XXIX. *On the Deflection of the Plumb-line in India, caused by the Attraction of the Himmalaya Mountains and of the elevated regions beyond; and its modification by the compensating effect of a Deficiency of Matter below the Mountain Mass. By the Venerable* JOHN HENRY PRATT, *M.A., Archdeacon of Calcutta. Communicated by Professor* STOKES, *Sec. R.S.*

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CONTENTS.

§ 1. *Reference to a former Paper on this subject.*

1. Two notices* which appeared last year in the Journal of the Astronomical Society on my Paper on Himmalayan Attraction, written at the Cape of Good Hope in 1854, and published by the Royal Society the year following, have called my attention again to this subject. Those who read that paper will remember, that it consisted of two parts; the first a calculation of the amount of deflection of the plumb-line, caused by the Mountain Mass in India, at the principal stations of the northern part of the Great Indian Arc; and the second, the effect which the application of these deflections, as corrections to the astronomical amplitudes, would have upon the calculated ellipticity of the Indian Arc. The results I arrived at are much greater than were anticipated. The author of the communications to the Astronomical Society proposes to test the truth of

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I am indebted to Mr. TENNANT for having detected a numerical error in page 98 of my paper. By going through the calculations, in page 98, it will be seen that

$$\text{for } \alpha = -0{\cdot}0039737 - 0{\cdot}0051426u + 0{\cdot}0016881v,$$
$$\text{we must read } \alpha = -0{\cdot}0019203 + 0{\cdot}0059576u - 0{\cdot}0014564v.$$

This will change the value of $a(1+\alpha)$ in the next line but one. These corrections have no effect, however, upon the results of my paper. When the paper was written, I was far away from all means of employing a computer, as is usual in such cases, to verify the long numerical calculations, not one-tenth of which appears in what is printed. In the last section of the present communication I have given a revise of such parts of the former one as need correction.

my results, by comparing the curvature thus deduced with the curvature of other arcs on the continent of India. But this proceeds upon the gratuitous hypothesis, and one which for geological reasons is most likely not true, that the earth is at present an exact spheroid of revolution; *i. e.* that all meridians are ellipses, and indeed the same ellipses, and that every arc of longitude is circular. There are only two ways of avoiding the conclusion regarding the curvature of the Indian Arc to which I came in my paper of 1855; either by showing that my data and reasoning are wrong, or by pointing out that some other cause is in operation, which either in whole or in part counteracts the effect of the Himmalayan Mass. My calculation has been before the public three years; and, though some small numerical errors have been detected, they are not of sufficient importance to affect the result; and the data I have every reason for believing to be correctly taken, as the Surveyor-General—who first called my attention to the subject in 1852, as an unsolved difficulty in the operations of the Great Trigonometrical Survey of India—has been requested to forward to me any corrections which may appear to him to be advisable, and none have been sent*. There remains, then, only the resource of looking for some counteracting cause to compensate for the large disturbance produced by the Himmalayas and the regions beyond.

2. The Astronomer Royal, in a paper published in the Transactions for 1855, suggested that immediately beneath the mountain-mass there was most probably a deficiency of matter, which would produce, as it were, a negative attraction, and so counteract the effect on the plumb-line. This hypothesis appears, however, to be

* The whole Mountain Region (which I call the Enclosed Space, see par. 8) I divide into two portions by a circular arc of about 350 miles radius described about Kaliana, the northern station of the Great Arc, as centre. The portion of the mountain country within that arc I have called, in my former Paper, the *Known Region*, because the heights are all readily obtained from the Survey Maps. The remaining portion I designate the *Doubtful Region*, because the heights cannot possibly be so well determined. My chief source of information for the Doubtful Region is HUMBOLDT's 'Aspects of Nature.' The Doubtful Region is, in superficial extent, about sixteen times as large as the Known Region; but, as I show in my former Paper, being more distant, produces nothing like a corresponding effect on the stations of the Arc.

By the use of the Tables in paragraphs 8 and 11 it may be shown without much difficulty, that the three Deflections $27''\cdot973$, $12''\cdot047$, $6''\cdot790$ at the three principal stations would be reduced only to $24''\cdot633$, $9''\cdot937$, $5''\cdot010$, if the whole of the Doubtful Region beyond a radius of about 700 miles from the northern station were left out of the reckoning. If the *whole* Doubtful Region were considered to be a dead flat and to have no effect at all—an hypothesis clearly impossible—the Deflections would still be reduced only to $12''\cdot972$, $3''\cdot219$, $1''\cdot336$. In these three cases the corrections to the amplitudes would be

$15''\cdot926$ and $5''\cdot257$ if the Doubtful Region be as I have taken it;
$14''\cdot696$ and $4''\cdot927$ if all beyond 700 miles from Kaliana be left out or annihilated;
$9''\cdot753$ and $1''\cdot883$ if the whole Doubtful Region be supposed non-existent.

The errors to be accounted for (if the ellipticity of the Indian Arc be what Colonel EVEREST assumes it to be, viz. the *mean*) are $5''\cdot236$ and $-3''\cdot789$. This extravagant, and indeed impossible hypothesis, then, of the non-existence of the vast Mountain Region beyond the Himmalaya crest, will not account for these errors. Much less, then, will any *mere correction* of the heights of this Doubtful Region. A solution, if there be one, *must* be sought for in another direction.

untenable for three reasons:—(1) It supposes the thickness of the earth's solid crust to be considerably smaller than that assigned by the only satisfactory physical calculations made on the subject—those by Mr. HOPKINS of Cambridge. He considers the thickness to be about 800 or 1000 miles at least. (2) It assumes that this thin crust is lighter than the fluid on which it is supposed to rest. But we should expect that in becoming solid from the fluid state, it would contract by loss of heat and become heavier. (3) The same reasoning by which Mr. AIRY makes it appear that every protuberance outside this thin crust must be accompanied by a protuberance inside, down into the fluid mass, would equally prove that wherever there was a hollow, as in deep seas, in the outward surface, there must be one also in the inner surface of the crust corresponding to it; thus leading to a law of varying thickness which no process of cooling could have produced.

§ 2. *Hypothesis of Deficiency of Matter adopted in this Paper.*

3. It is nevertheless to this source—I mean a Deficiency of Matter below—that we must look, I feel fully assured, for a compensating cause, if any is to be found. My present object is to propose another hypothesis regarding deficiency of matter below the mountain-mass, as first suggested by Mr. AIRY; and to reduce my hypothesis to the test of calculation.

I will here observe, that all the more laborious numerical calculations in this Paper have been performed for me—as I could not possibly find leisure for the work—by a practised Computer of the Great Trigonometrical Survey Office in Calcutta, obligingly recommended to me by the Deputy Surveyor-General. In the course of the present paper the attraction of the Mountain Mass, in the direction of the meridian, at the three principal stations of the Great Arc—Kaliana, Kalianpur, and Damargida—are again calculated, and a test so far afforded of the general accuracy of my former results.

4. I will now state the hypothesis on which my present calculation proceeds. At the time when the earth had just ceased to be wholly fluid, the form must have been a perfect spheroid, with no mountains and valleys nor ocean-hollows. As the crust formed, and grew continually thicker, contractions and expansions may have taken place in any of its parts, so as to depress and elevate the corresponding portions of the surface. If these changes took place chiefly in a vertical direction, then at any epoch a vertical line drawn down to a sufficient depth from any place in the surface will pass through a mass of matter which has remained the same in amount all through the changes. By the process of expansion the mountains have been forced up, and the mass thus raised above the level has produced a corresponding *attenuation* of matter below. This attenuation is most likely very trifling, as it probably exists through a great depth. Whether this cause will produce a sufficient amount of compensation can be determined only by submitting it to calculation, which I proceed to do.

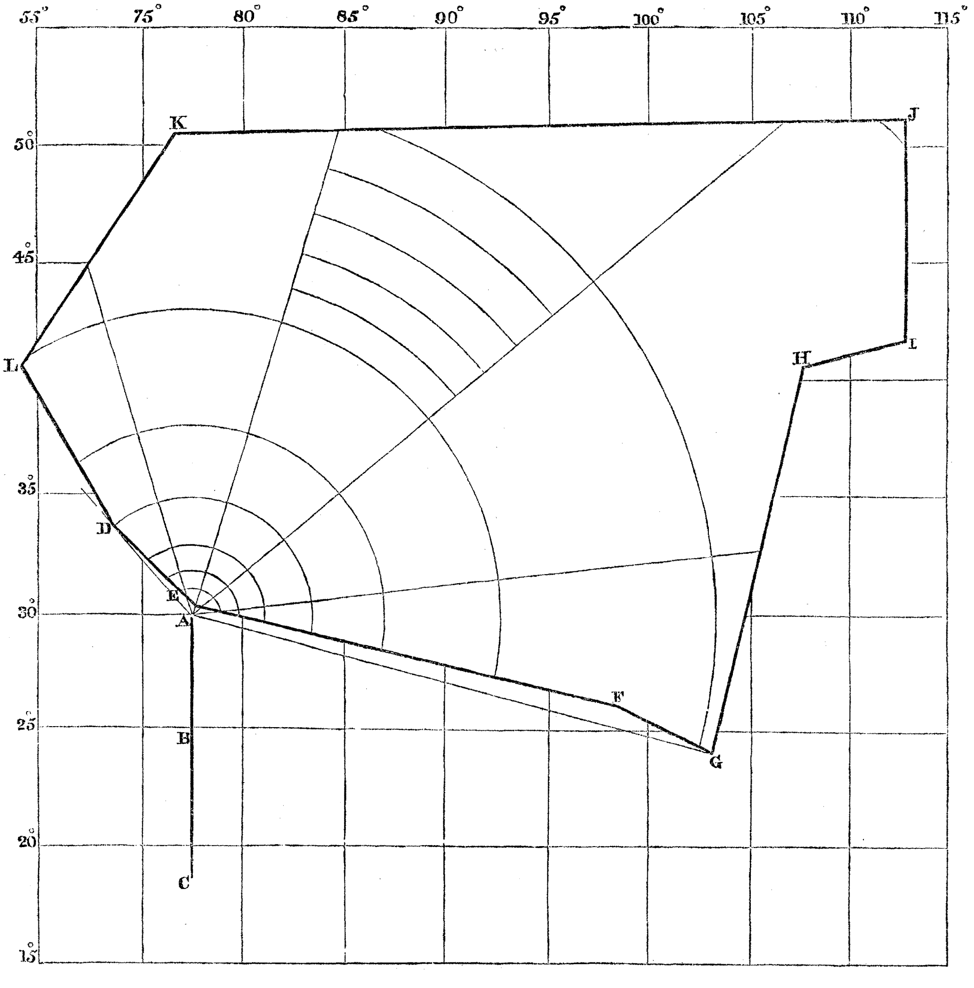
§ 3. *Summary of data and of some of the results of the former Paper, necessary for the present calculation.*

5. Before proceeding it is necessary here to gather into one view such of the results of my former Paper* as will be required for the present calculation.

The Great Arc under consideration is in longitude 77° 42′. The three principal stations of this arc, the latitudes of which have been observed astronomically, are A, or Kaliana, in latitude 29° 30′ 48″; B, or Kalianpur, 24° 7′ 11″; and C, or Damargida, 18° 3′ 15″. The lengths of the two arcs, Kaliana—Kalianpur, and Kalianpur—Damargida, as determined by the Survey, are 326859·52 and 367154·37 fathoms.

6. In order to effect the calculation of the attraction on the three stations, the mass is conceived to be divided into smaller masses in the following manner, taking the station A for example. From the point A (fig. 1) great circles are drawn meeting in

Fig. 1.



* Philosophical Transactions, 1855, pp. 53–100.

the antipodes of A, dividing the superficial mass into so many lunes of attracting matter. Circles are then drawn with their centres in the vertical line through A, at distances increasing according to a peculiar law, which, for a name, I call the Law of Dissection, dividing the whole surface into a number of four-sided "compartments*," which have this property, that the attractions of the masses standing on the several compartments of a lune thus formed are exactly the same when the heights of the masses are the same; or, which amounts to the same thing, since the heights are all small compared with the distances from the station, the attractions of the masses standing upon the several compartments are in proportion to their average heights.

7. It is then proved (p. 65) that if β be the angular width of the compartment, and h the average height above the level of A, the deflection of the plumb-line at A, in the direction of the meridian, caused by the mass standing on this compartment,

$$=1''\cdot1392\, h \sin\tfrac{1}{2}\beta \cos \text{Az}.$$

Hence the deflection caused by the whole mass

$$=1''\cdot1392\, \Sigma.h \sin\tfrac{1}{2}\beta \cos \text{Az}, \qquad (1.)$$

Az being the azimuth, reckoned from the north, of the middle line of the lune.

8. By an examination of the physical geography of the earth's surface, I next show that the only mass which can affect the plumb-line lies within a space DEFGHIJKLD, which I call the Enclosed Space, and then describe: the extreme diagonal lengths are each about 2000 miles.

Tables are next formed—of which a summary is given in the following page—containing all the data regarding the average heights of the masses on the several compartments; and the formula (1.) brings out the following results:—

	At A.	At B.	At C.
Deflection of the plumb-line in the meridian . .	$27''\cdot853$	$11''\cdot968$	$6''\cdot909$

Hence the differences of deflections, or the errors in the astronomical amplitudes, are

$$15''\cdot885 \text{ and } 5''\cdot059.$$

After applying these corrections, I proceed to calculate the ellipticity of the Indian Arc. With a view to make the results available for any other calculated attractions—a

* The horizontal attraction of the mass standing on any one of these compartments is shown to equal (see p. 62 of former Paper)

$$\rho \sin\frac{1}{2}\beta \frac{\phi \cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)} h,$$

where α and $\alpha+\phi$ are the angular distances of the nearer and further sides of the compartment from A. The Law of Dissection, connecting ϕ with α, is this,

$$\frac{\phi \cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}=\frac{21}{4},$$

ϕ being assumed equal to $\frac{1}{10}\alpha$ when these angles are very small.

TABLE of the average HEIGHTS (in feet) of the masses in the several compartments above the level of Kaliana.

No. of compartment.	Distance of middle of compartment from station.	For Station A (Kaliana).					For Station B (Kalianpur).					For Station C (Damargida).		
		Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.	Lune I.	Lune II.	Lune III.
1	0°·787	......	......	......	600	......	......	......	......	......	......	......	......	......
2	0·866	......	......	500	600	......	......	......	......	......	......	......	......	......
3	0·949	......	......	600	1,200	300	......	......	......	......	......	......	......	......
4	1·048	......	500	1,500	800	300	......	......	......	......	......	......	......	......
5	1·153	300	800	1,500	1,500	300	......	......	......	......	......	......	......	......
6	1·268	500	1,000	1,200	1,600	300	......	......	......	......	......	......	......	......
7	1·395	800	2,000	600	1,600	800	......	......	......	......	......	......	......	......
8	1·534	1,300	3,000	2,000	1,800	1,000	......	......	......	......	......	......	......	......
9	1·687	2,300	3,000	2,500	2,500	1,500	......	......	......	......	......	......	......	......
10	1·856	1,800	5,000	3,000	6,000	2,000	......	......	......	......	......	......	......	......
11	2·042	300	2,500	5,500	7,000	2,500	......	......	......	......	......	......	......	......
12	2·247	800	4,000	7,000	7,000	3,000	......	......	......	......	......	......	......	......
13	2·472	800	4,000	8,500	9,000	2,500	......	......	......	......	......	......	......	......
14	2·719	800	5,000	9,500	9,000	2,500	......	......	......	......	......	......	......	......
15	2·990	800	5,500	9,000	9,000	2,500	......	......	......	......	......	......	......	......
16	3·289	800	6,000	9,500	9,000	2,500	......	......	......	......	......	......	......	......
17	3·616	1,300	7,000	9,500	9,500	2,500	......	......	......	......	......	......	......	......
18	3·980	1,800	7,500	9,500	9,500	3,000	......	......	......	......	......	......	......	......
19	4·378	2,800	8,300	9,500	9,500	3,000	......	......	......	......	......	......	......	......
20	4·813	4,800	9,500	9,500	9,500	3,000	......	......	......	......	......	......	......	......
21	5·298	7,000	8,700	8,500	7,000	3,000	......	......	......	......	......	......	......	......
22	5·828	8,000	8,200	8,000	9,000	3,750	......	......	500	......	......	......	......	......
23	6·408	9,000	7,700	7,500	8,600	4,500	......	500	2,500	......	......	......	......	......
24	7·054	9,000	7,200	7,000	8,100	5,250	......	2,500	7,000	......	......	......	......	......
25	7·707	8,700	6,800	6,500	7,600	6,000	......	4,500	9,000	......	......	......	......	......
26	8·533	8,400	6,300	6,000	7,100	6,750	......	6,500	9,500	......	......	......	......	......
27	9·386	8,100	5,800	5,500	6,700	7,500	1,200	8,200	9,000	4,000	......	......	......	......
28	10·324	7,900	5,400	5,000	6,200	8,250	6,000	9,500	8,250	5,700	......	......	......	......
29	11·360	7,600	4,900	4,500	5,700	9,000	9,000	9,000	7,500	7,300	......	......	......	......
30	12·506	7,300	4,400	4,000	5,200	9,000	9,000	8,200	6,750	9,000	......	600	......	......
31	13·770	7,000	3,900	3,500	4,800	8,250	8,450	7,400	6,000	9,000	......	4,000	6,000	......
32	15·211	......	3,450	3,000	4,300	7,500	7,850	6,600	5,250	8,150	5,000	7,300	9,000	......
33	16·80	......	3,000	2,500	3,850	6,750	7,250	5,750	4,500	7,300	7,000	9,500	7,950	7,000
34	18·51	......	2,500	2,000	3,400	6,000	6,700	4,900	3,750	6,450	9,000	9,000	6,850	9,000
35	20·40	......	2,000	1,500	2,900	......	......	4,100	3,000	5,550	8,250	7,900	5,800	8,100
36	22·50	......	......	1,500	2,400	......	......	3,300	2,250	4,700	7,500	6,850	4,700	7,250
37	24·83	......	......	3,500	1,900	......	......	2,500	1,500	3,850	......	5,750	3,650	6,400
38	27·43	......	......	......	1,500	......	......	......	2,000	3,000	......	4,650	2,550	5,500
39	30·31	......	......	......	1,500	......	......	......	3,000	......	......	3,600	1,500	......
40	33·55	......	......	......	4,000	......	......	......	......	......	......	2,500	2,000	......
Totals in feet		110,000	154,850	180,400	207,950	125,000	55,450	83,450	91,250	74,000	36,750	61,650	50,000	43,250
Totals in miles		20·833	29·328	34·167	39·385	23·673	10·502	15·805	17·282	14·015	6·960	11·676	9·470	8·191

precaution I now feel the value of—I multiply them by arbitrary coefficients $1-u$ and $1-v$, making them

$$15''\cdot885\,(1-u) \text{ and } 5''\cdot059\,(1-v).$$

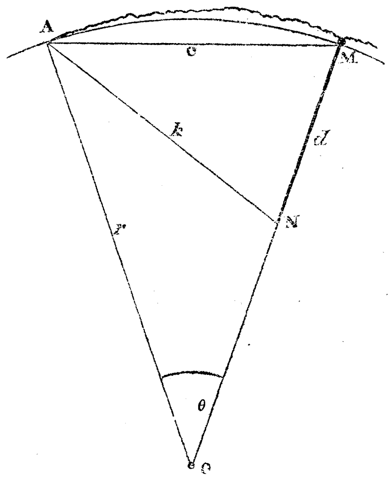
The ellipticity and the semi-axis major then are

$$\left.\begin{aligned} \varepsilon &= 0\cdot002346 + 0\cdot003693u - 0\cdot001046\,v \\ a &= 2088274\cdot3 + 0\cdot0059576u - 0\cdot0014564v \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

These are the only results which it is necessary here to recapitulate.

§ 4. *Calculation of the Effect of Deficiency of Matter, according to the assumed hypothesis.*

Fig. 2.



9. I now proceed to calculate the effect of the attenuation caused below by the upheaval of the mountains. I shall suppose that the masses standing upon the several compartments are concentrated into the middle points of the meridian-diameters of the compartments. This may be done without sensible error, as shown in the note*. And next that these masses are distributed into vertical bars or prisms, all to the same depth *d*, each bar or prism being of uniform density in itself, but containing the whole amount of matter belonging to the compartment from the centre of which it is drawn down. The difference between the attractions of the mass thus distributed downwards, and of the mass as it lies in the Himmalayas, will give the horizontal attraction on the plumb-line at the three stations A, B, C, on the hypothesis of the attenuation I have described.

10. Let M (fig. 2) be one of these masses, concentrated in the middle of its compartment: MN the bar or prism into which it is distributed. Join AM, AN, AO, NO. Let $AM=c$, $AN=k$, $AO=r$, $MN=d$, $\angle AOM=\theta$. It is a property easily proved, that the attraction of the prism MN on A in the direction $AM=\frac{M}{c.k}$. But the attraction of M on A in the same direction $=\frac{M}{c^2}$. The ratio of these is $\frac{c}{k}$. In this ratio, therefore, must the attractions of the masses on the compartments, as deduced in my former Paper (and re-calculated in this), be reduced, to obtain from them the attractions of the same masses when distributed into vertical prisms.

∴ Deflection along the middle line of any lune, caused by the mass on the compartment when distributed down to a depth *d*, becomes

$$=1''\cdot 1392\, h\frac{c}{k}\sin\frac{1}{2}\beta\,;$$

$$\therefore \text{ deflection in the meridian } =1''\cdot 1392\, h\frac{c}{k}\sin\frac{1}{2}\beta\cos \text{Az}.$$

* Area of compartment $=r^2\beta\left(\cos\alpha-\cos(\alpha+\phi)\right)$, r=rad. of earth.

Mass on the compartment $=r^2\beta\rho\left(\cos\alpha-\cos(\alpha+\phi)\right)h=4r^2\beta\rho h\sin\frac{\varphi}{2}\sin\left(\frac{\alpha}{2}+\frac{\varphi}{4}\right)\cos\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)$,

$$\therefore \frac{\text{attraction of the mass, concentrated in the centre}}{\text{ditto (from last note)}}=\frac{\frac{1}{2}\beta}{\sin\frac{1}{2}\beta}\;\frac{\sin\frac{1}{2}\varphi}{\frac{1}{2}\varphi}=1+\frac{\beta^2-\varphi^2}{24}+\ldots$$

The largest value of this is when $\beta=\frac{30}{180}\pi=\frac{11}{21}$, and φ is evanescent, when it is about $\frac{1}{100}$th part greater than unity.

If Σ be a symbol representing the sum of the quantities $h \sin \frac{1}{2} \beta \cos \text{Az}$ for the five lunes for any given value of $\frac{c}{k}$, and $\Sigma . h \sin \frac{1}{2} \beta \cos \text{Az} = \text{S}$,

$\therefore$ Deflection in the meridian, for these five compartments, $= 1''{\cdot}1392 \frac{c}{k} \text{S}$. . . . (3.)

11. We must first obtain the values of S. The width β of the several lunes into which the Enclosed Space is divided, and the azimuth of the middle lines, are as follows (see par. 39 of my former Paper):—for A, widths, 24°, 30°, 30°, 30°, 20°, azimuths, 27°, 0°, 30°, 60°, 85°; for B, widths, 8°, 30°, 30°, 30°, 10°, azimuths, 19°, 0°, 30°, 60°, 80°; for C, widths, 30°, 30°, 20°, azimuths, 0°, 30°, 55°. Hence the values of $\sin \frac{1}{2} \beta \cos \text{Az}$ for the several lunes are—

for A, 0·1852, 0·2588, 0·2241, 0·1294, 0·0151;
for B, 0·0660, 0·2588, 0·2241, 0·1294, 0·0151;
for C, 0·2588, 0·2241, 0·0996.

By these numbers must the heights in the Table of Heights (page 750) be multiplied; they must then be added together from left to right, and the totals, given in the Table in page 753, will be the aggregates for the several distances from the station, that is, the values of S. The grand totals at the foot of the Table, reduced to miles and multiplied by $1''{\cdot}1392$, give the total meridian deflections at the three stations caused by the mountain-mass as it now lies at the surface. These differ but slightly from the results of my last Paper, as recapitulated in par. 22 of the present communication *.

12. The values of $\frac{c}{k}$ have now to be determined, in order to carry out the calculation of formula (3.). By fig. 2 we have

$$k^2 = c^2 + d^2 - 2cd \sin \frac{1}{2} \theta, \qquad \left(c = 2a \sin \frac{1}{2} \theta\right)$$

$$= 4a^2 \sin^2 \frac{1}{2} \theta + d^2 - 4ad \sin^2 \frac{1}{2} \theta = d^2 + 4a^2 \sin^2 \frac{\theta}{2} \left(1 - \frac{d}{a}\right).$$

Put $$\frac{2a}{d} \sqrt{1 - \frac{d}{a}} \sin \frac{1}{2} \theta = \tan \psi, \qquad \therefore k^2 = d^2 \sec^2 \psi,$$

$$\therefore \frac{c}{k} = \frac{2a}{d} \cos \psi \sin \frac{1}{2} \theta.$$

These formulæ give

$$\log \frac{2a}{d} \sqrt{1 - \frac{d}{a}} + \log \sin \frac{1}{2} \theta = \log \tan \psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

$$\log \frac{2a}{d} + \log \cos \psi + \log \sin \frac{1}{2} \theta - 20 = \log \frac{c}{k} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

* To test the accuracy of these results, the totals of the thirteen columns of the Table in page 750 have been multiplied by the thirteen constants above given, and then the three grand totals taken for the three stations. They are found to accord exactly with the three totals of the Table in page 753.

TABLE OF DEFLECTIONS in Meridian (*mass all at the surface*) (from formula 3).

No. of compartment.	Values of $S=\Sigma.h \sin\frac{1}{2}\beta \cos \text{Az.}$		
	Station A. Kaliana.	Station B. Kalianpur.	Station C. Damargida.
1	77·64	....	....
2	189·69	....	....
3	294·27	....	....
4	573·60	....	....
5	797·38	....	....
6	831·89	....	....
7	1019·34	....	....
8	1713·38	....	....
9	2108·76	....	....
10	3106·26	....	....
11	2878·66	....	....
12	3703·16	....	....
13	4290·56	....	....
14	4773·46	....	....
15	4790·81	....	....
16	5032·26	....	....
17	5448·36	....	....
18	5677·91	....	....
19	6070·15	....	....
20	6751·11	....	....
21	6403·91	....	....
22	6617·79	112·05	....
23	6521·10	689·65	....
24	6225·27	2215·70	....
25	5901·77	3181·50	....
26	5551·39	3811·15	....
27	5213·94	4735·86	....
28	4907·95	5441·00	....
29	4557·57	5548·57	....
30	4195·86	5393·44	155·28
31	3835·76	4982·02	2379·80
32	2234·83	4532·81	3906·14
33	1936·77	4025·37	4937·39
34	1625·76	3521·23	4760·69
35	1229·01	2576·13	4151·06
36	646·71	2079·69	3547·65
37	1030·21	1481·34	2943·01
38	194·10	836·40	2322·17
39	194·10	672·30	1267·83
40	517·60	....	1095·20
In feet	129670·05	55836·21	31466·22
In miles	24·559	10·575	5·960
Multiplying by 1″·1392, DEFLECTIONS=27″·978		12″·047	6″·790

The Tables in the following pages (which I call the Four TABLES OF REDUCTION), give the values of $\frac{c}{k}$ for four values of the depth, viz. $d=100$, 300, 500, and 1000 miles. For these values of d, the two constants $\log\frac{2a}{d}\sqrt{1-\frac{d}{a}}$ and $\log\frac{2a}{d}$ are respectively

Four Tables of Reduction, calculated by formulæ (4.) and (5.).

Table I.—Depth =100 miles.

No. of the compartment.	$\frac{1}{2}\theta$.	$\log\sin\frac{1}{2}\theta$.	$\log\sin\frac{1}{2}\theta$ $+1\cdot8975923$ $=\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+1\cdot9030900-20$ $=\log\frac{c}{k}$.
1	0°·393 or 0° 23′ 35″	7·8363279	9·7339202	9·9440904	$\bar{1}$·6835083
2	0·433 or 0 25 59	7·8784168	9·7760091	9·9337984	$\bar{1}$·7152952
3	0·474 or 0 28 26	7·9175489	9·8151412	9·9228096	$\bar{1}$·7434485
4	0·524 or 0 31 26	7·9611105	9·8587028	9·9088382	$\bar{1}$·7730387
5	0·576 or 0 34 34	8·0023763	9·8999686	9·8937914	$\bar{1}$·7992677
6	0·634 or 0 38 2	8·0438816	9·9414739	9·8767823	$\bar{1}$·8237539
7	0·697 or 0 41 49	8·0850648	9·9826571	9·8579831	$\bar{1}$·8461379
8	0·767 or 0 46 1	8·1266283	10·0242206	9·8370370	$\bar{1}$·8667553
9	0·843 or 0 50 35	8·1677179	10·0653102	9·8143838	$\bar{1}$·8851917
10	0·928 or 0 55 41	8·2094324	10·1070247	9·7894454	$\bar{1}$·9024678
11	1·021 or 1 1 16	8·2509274	10·1485197	9·7627636	$\bar{1}$·9167810
12	1·123 or 1 7 23	8·2922508	10·1898431	9·7344460	$\bar{1}$·9297868
13	1·236 or 1 14 10	8·3339012	10·2314935	9·7042486	$\bar{1}$·9412398
14	1·359 or 1 21 26	8·3744877	10·2720800	9·6733584	$\bar{1}$·9509361
15	1·495 or 1 29 42	8·4164693	10·3140616	9·6400279	$\bar{1}$·9595872
16	1·644 or 1 38 38	8·4576902	10·3552825	9·6060825	$\bar{1}$·9668627
17	1·808 or 1 48 29	8·4990171	10·3966094	9·5709770	$\bar{1}$·9730841
18	1·990 or 1 59 24	8·5406431	10·4382354	9·5346689	$\bar{1}$·9784020
19	2·189 or 2 11 20	8·5819954	10·4795877	9·4977795	$\bar{1}$·9828649
20	2·406 or 2 24 22	8·6230654	10·5206577	9·4604449	$\bar{1}$·9869003
21	2·649 or 2 38 56	8·6647864	10·5623787	9·4219106	$\bar{1}$·9897870
22	2·914 or 2 54 50	8·7061631	10·6037554	9·3831792	$\bar{1}$·9924323
23	3·204 or 3 12 14	8·7473285	10·6449208	9·3442061	$\bar{1}$·9946246
24	3·527 or 3 31 37	8·7890017	10·6865940	9·3044808	$\bar{1}$·9965725
25	3·853 or 3 51 11	8·8273552	10·7249475	9·2674791	$\bar{1}$·9979243
26	4·266 or 4 15 58	8·8715081	10·7691004	9·2246997	$\bar{1}$·9992978
27	4·693 or 4 41 35	8·9128474	10·8104397	9·1844141	0·0003515
28	5·162 or 5 9 43	8·9541030	10·8516953	9·1440456	0·0012386
29	5·680 or 5 40 48	8·9955141	10·8931064	9·1033699	0·0019740
30	6·253 or 6 15 11	9·0371072	10·9346995	9·0623859	0·0025831
31	6·885 or 6 53 6	9·0787356	10·9763279	9·0212643	0·0030899
32	7·605 or 7 36 18	9·1217006	11·0192929	8·9787284	0·0035190
33	8·40 or 8 24 0	9·1645998	11·0621921	8·9361833	0·0038731
34	9·25 or 9 15 0	9·2061309	11·1037232	8·8949442	0·0041651
35	10·20 or 10 12 0	9·2481811	11·1457734	8·8531198	0·0043909
36	11·25 or 11 15 0	9·2902357	11·1878280	8·8112590	0·0045847
37	12·41 or 12 24 36	9·3322481	11·2298404	8·7694070	0·0047451
38	13·71 or 13 42 36	9·3747625	11·2723548	8·7270362	0·0048887
39	15·15 or 15 9 0	9·4172174	11·3148097	8·6846817	0·0049891
40	16·77 or 16 46 12	9·4601918	11·3577841	8·6417987	0·0050805

Four Tables of Reduction, calculated by formulæ (4.) and (5.).

No. of the compartment.	Table II.—Depth =300 miles.			Table III.—Depth =500 miles.		
	$\log\sin\frac{1}{2}\theta$ $+1\cdot4090396$ $=\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+1\cdot4259687-20$ $=\log\frac{c}{k}$.	$\log\sin\frac{1}{2}\theta$ $+1\cdot1751240$ $=\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+1\cdot2041200-20$ $=\log\frac{c}{k}$.
1	9·2453674	9·9933804	$\bar{1}$·2556770	9·0114519	9·9977228	$\bar{1}$·0381707
2	9·2874563	9·9919899	$\bar{1}$·2963754	9·0535408	9·9972390	$\bar{1}$·0797758
3	9·3265884	9·9904426	$\bar{1}$·3339604	9·0926729	9·9966980	$\bar{1}$·1183669
4	9·3701500	9·9883756	$\bar{1}$·3754548	9·1362345	9·9959712	$\bar{1}$·1612017
5	9·4114158	9·9860193	$\bar{1}$·4143643	9·1775003	9·9951373	$\bar{1}$·1016336
6	9·4529211	9·9831858	$\bar{1}$·4530361	9·2190056	9·9941268	$\bar{1}$·2421284
7	9·4941043	9·9798329	$\bar{1}$·4908664	9·2601888	9·9929198	$\bar{1}$·2821046
8	9·5356678	9·9758083	$\bar{1}$·5284053	9·3017523	9·9914554	$\bar{1}$·3222037
9	9·5767574	9·9710902	$\bar{1}$·5647768	9·3428419	9·9897169	$\bar{1}$·3615548
10	9·6184719	9·9654319	$\bar{1}$·6008330	9·3845564	9·9893680	$\bar{1}$·4029204
11	9·6599669	9·9588048	$\bar{1}$·6357009	9·4260514	9·9850772	$\bar{1}$·4401246
12	9·7012903	9·9510786	$\bar{1}$·6692981	9·4673748	9·9820752	$\bar{1}$·4784460
13	9·7429407	9·9420082	$\bar{1}$·7018781	9·5090252	9·9784675	$\bar{1}$·5164887
14	9·7835272	9·9317948	$\bar{1}$·7322512	9·5496117	9·9742951	$\bar{1}$·5529028
15	9·8255088	9·9196563	$\bar{1}$·7620943	9·5915933	9·9691849	$\bar{1}$·5897742
16	9·8667297	9·9060534	$\bar{1}$·7897123	9·6328142	9·9632611	$\bar{1}$·6250713
17	9·9080566	9·8906270	$\bar{1}$·8156128	9·6741411	9·9562887	$\bar{1}$·6594258
18	9·9476826	9·8731890	$\bar{1}$·8398008	9·7157671	9·9480801	$\bar{1}$·6928432
19	9·9910349	9·8539217	$\bar{1}$·8618858	9·7571194	9·9386035	$\bar{1}$·7247189
20	10·0321049	9·8328399	$\bar{1}$·8818740	9·7981894	9·9277437	$\bar{1}$·7549291
21	10·0738259	9·8094513	$\bar{1}$·9002064	9·8399104	9·9150995	$\bar{1}$·7840059
22	10·1152026	9·7843319	$\bar{1}$·9164637	9·8812871	9·9008277	$\bar{1}$·8111108
23	10·1563680	9·7575188	$\bar{1}$·9308160	9·9224525	9·8848149	$\bar{1}$·8362634
24	10·1980412	9·7286294	$\bar{1}$·9435998	9·9641257	9·8666831	$\bar{1}$·8598048
25	10·2363947	9·7005925	$\bar{1}$·9539164	10·0024792	9·8482392	$\bar{1}$·8797144
26	10·2805476	9·6667435	$\bar{1}$·9642203	10·0466321	9·8249209	$\bar{1}$·9005490
27	10·3218869	9·6336708	$\bar{1}$·9724869	10·0879714	9·8010778	$\bar{1}$·9180452
28	10·3631425	9·5994871	$\bar{1}$·9795588	10·1292270	9·7753959	$\bar{1}$·9336189
29	10·4045536	9·5641182	$\bar{1}$·9856010	10·1706381	9·7478206	$\bar{1}$·9474547
30	10·4461467	9·5276703	$\bar{1}$·9907462	10·2122312	9·7184113	$\bar{1}$·9596385
31	10·4877751	9·4903904	$\bar{1}$·9950947	10·2538596	9·6873971	$\bar{1}$·9702527
32	10·5307401	9·4511894	$\bar{1}$·9988587	10·2968246	9·6538752	$\bar{1}$·9796958
33	10·5736393	9·4174137	0·0019822	10·3397238	9·6190275	$\bar{1}$·9877473
34	10·6151704	9·3724169	0·0045165	10 3812549	9·5841387	$\bar{1}$·9943896
35	10·6572206	9·3315394	0·0056892	10·4233051	9·5477953	0·0000964
36	10·6992752	9·2922210	0·0084254	10·4653597	9·5105573	0·0049130
37	10·7412876	9·2516772	0·0098940	10·5073721	9·4725938	0·0089619
38	10·7838020	9·2104015	0·0111327	10·5498865	9·4335103	0·0123928
39	10·8262569	9·1689547	0·0121408	10·5923414	9·3939077	0·0152451
40	10·8692313	9·1268419	0·0130024	10·6353158	9·3533447	0·0176565

Four Tables of Reduction, calculated by formulæ (4) and (5).

Table IV.—Depth = 1000 miles.

No. of the compartment.	$\frac{1}{2}\theta$.	$\log \sin \frac{1}{2}\theta$.	$\log \sin \frac{1}{2}\theta + 0\cdot 8406207 = \log \tan \psi$.	$\log \cos \psi$.	$\log \sin \frac{1}{2}\theta + \log \cos \psi + 0\cdot 9030900 - 20 = \log \frac{c}{k}$.
1	$0^\circ\cdot 393$ or $0^\circ\ 23'\ 35''$	7·8363279	8·6769486	9·9995100	$\bar{2}$·7389279
2	0·433 or 0 25 59	7·8784168	8·7190375	9·9994054	$\bar{2}$·7809122
3	0·474 or 0 28 26	7·9175489	8·7581696	9·9992882	$\bar{2}$·8209271
4	0·524 or 0 31 26	7·9611105	8·8017312	9·9991303	$\bar{2}$·8633308
5	0·576 or 0 34 34	8·0023763	8·8429970	9·9989487	$\bar{2}$·9044150
6	0·634 or 0 38 2	8·0438816	8·8845023	9·9987280	$\bar{2}$·9456996
7	0·697 oa 0 41 49	8·0850648	8·9256855	9·9984634	$\bar{2}$·9866182
8	0·767 or 0 46 1	8·1266283	8·9672490	9·9981406	$\bar{1}$·0278589
9	0·843 or 0 50 35	8·1677179	9·0083386	9·9977552	$\bar{1}$·0685631
10	0·928 or 0 55 41	8·2094324	9·0500531	9 9972826	$\bar{1}$·1098050
11	1·021 or 1 1 16	8·2509274	9·0915481	9·9967147	$\bar{1}$·1507321
12	1·123 or 1 7 23	8·2922508	9·1328715	9·9960323	$\bar{1}$·1913731
13	1·236 or 1 14 10	8·3339012	9·1745219	9·9952026	$\bar{1}$·2321938
14	1·359 or 1 21 26	8·3744877	9·2151084	9·9942299	$\bar{1}$·2718076
15	1·495 or 1 29 42	8·4164693	9·2570900	9·9930188	$\bar{1}$·3125781
16	1·644 or 1 38 38	8·4576902	9·2983109	9·9915873	$\bar{1}$·3523675
17	1·808 or 1 48 29	8·4990171	9·3396378	9·9898643	$\bar{1}$·3919714
18	1·990 or 1 59 24	8·5406431	9·3812638	9·9877819	$\bar{1}$·4315150
19	2·189 or 2 11 20	8·5819954	9·4226161	9·9853037	$\bar{1}$·4703891
20	2·406 or 2 24 22	8·6230654	9·4636861	9·9823649	$\bar{1}$·5085203
21	2·649 or 2 38 56	8·6647864	9·5054071	9·9788067	$\bar{1}$·5466831
22	2·914 or 2 54 50	8·7061631	9·5467838	9·9746090	$\bar{1}$·5838621
23	3·204 or 3 12 14	8·7473285	9·5879492	9·9696639	$\bar{1}$·6200824
24	3·527 or 3 31 37	8·7890017	9·6296224	9·9637547	$\bar{1}$·6558464
25	3·853 or 3 51 11	8·8273552	9·6679759	9·9574003	$\bar{1}$·6878455
26	4·266 or 4 15 58	8·8715081	9·7121288	9·9488494	$\bar{1}$·7234475
27	4·693 or 4 41 35	8·9128474	9·7534681	9·9394965	$\bar{1}$·7554339
28	5·162 or 5 9 43	8·9541030	9·7947237	9·9287188	$\bar{1}$·7859118
29	5·680 or 5 40 48	8·9955141	9·8361348	9·9163146	$\bar{1}$·8149187
30	6·253 or 6 15 11	9·0371072	9·8777279	9·9021263	$\bar{1}$·8423235
31	6·885 or 6 53 6	9·0787356	9·9193563	9·8860837	$\bar{1}$·8679093
32	7·605 or 7 36 18	9·1217006	9·9623213	9·8674166	$\bar{1}$·8922072
33	8·40 or 8 24 0	9·1645998	10·0052205	9·8468584	$\bar{1}$·9145482
34	9·25 or 9 15 0	9·2061309	10·0467516	9·8248529	$\bar{1}$·9340738
35	10·20 or 10 12 0	9·2481811	10 0888018	9·8005771	$\bar{1}$·9518482
36	11·25 or 11 15 0	9·2902357	10·1308564	9·7743449	$\bar{1}$·9676706
37	12·41 or 12 24 36	9·3322481	10·1728688	9·7462853	$\bar{1}$·9816234
38	13·71 or 13 42 36	9·3747625	10·2153852	9·7161171	$\bar{1}$·9939696
39	15·15 or 15 9 0	9·4172174	10·2578381	9·6843535	0·0046609
40	16·77 or 16 46 12	9·4601918	10·3008125	9·6506910	0·0139728

TABLE OF DEFLECTIONS, for Station A (*Kaliana*), when the mountain-mass is distributed to the several depths specified.

No. of the compartment.	Depth = 100 miles.			Depth = 300 miles.		Depth = 500 miles.		Depth = 1000 miles.	
	log S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.
1	1·8900855	1·5735938	37·462	1·1457625	13·988	0·9282562	8·477	0·6290134	4·256
2	2·2780444	1·9933396	98·478	1·5744198	37·533	1·3578202	22·794	1·0589566	11·454
3	2·4687460	2·2121945	163·002	1·8027064	63·490	1·5871129	38·647	1·2896731	19·484
4	2·7586091	2·5316478	340·132	2·1340639	136·164	1·9198108	83·140	1·6219399	41·874
5	2·9016653	2·7009330	502·265	2·3160296	207·028	2·0032989	100·762	1·8060803	63·984
6	2·9200659	2·7438198	554·396	2·3731020	236·103	2·1621943	145·276	1·9657655	92·420
7	3·0083191	2·8544570	715·248	2·4991855	315·635	2·2904237	195·175	1·9949373	98·841
8	3·2338537	3·1006090	1260·692	2·7622590	578·441	2·5560574	359·797	2·2617126	182·689
9	3·3240271	3·2091188	1618·522	2·8888039	774·112	2·6855819	484·822	2·3925902	246·940
10	3·4922377	3·3947055	2481·450	3·0930707	1238·998	2·8951581	785·521	2·6020427	399·984
11	3·4591903	3·3759713	2376·683	3·0948912	1244·203	2·8993149	793·076	2·6099224	407·308
12	3·5685724	3·4983592	3150·360	3·2378705	1729·300	3·0470184	1114·346	2·7599455	575·368
13	3·6325039	3·5737437	3747·518	3·3343820	2159·643	3·1489926	1409·265	2·8646977	732·314
14	3·6788333	3·6297694	4263·530	3·4110845	2576·822	3·2317361	1705·046	2·9506409	892·567
15	3·6804089	3·6399961	4365·120	3·4425032	2770·150	3·2701831	1862·873	2·9929870	983·982
16	3·7017631	3·6686258	4661·519	3·4914754	3100·811	3·3268344	2122·435	3·0541306	1132·741
17	3·7362658	3·7093499	5120·942	3·5518786	3563·515	3·3956916	2487·090	3·1282372	1343·498
18	3·7541885	3·7325905	5424·447	3·5939893	3926·353	3·4470317	2799·186	3·1857035	1533·570
19	3·7831994	3·7660643	5835·315	3·6450852	4416·571	3·5079183	3220·463	3·2535885	1793·034
20	3·8293752	3·8162755	6550·515	3·7112492	5143·387	3·5843043	3839·761	3·3378955	2177·186
21	3·8064452	3·7962322	6255·070	3·7066516	5089·225	3·5904511	3894·494	3·3531283	2254·905
22	3·8207130	3·8131453	6503·470	3·7371767	5459·800	3·6318238	4283·747	3·4045751	2538·488
23	3·8143209	3·8089455	6440·884	3·7451369	5560·795	3·6505843	4472·850	3·4344033	2718·963
24	3·7941586	3·7907311	6176·339	3·7377584	5467·118	3·6539634	4507·787	3·4500050	2818·416
25	3·7709822	3·7689065	5873·630	3·7248986	5307·605	3·6506966	4474·006	3·4588277	2876·257
26	3·7444017	3·7436995	5542·420	3·7086220	5112·366	3·6449507	4415·203	3·4678492	2936·630
27	3·7171660	3·7175175	5218·151	3·6896529	4893·876	3·6352112	4317·290	3·4725999	2968·930
28	3·6909007	3·6921393	4921·973	3·6704595	4682·303	3·6245196	4212·303	3·4768125	2997·868
29	3·6587334	3·6607076	4578·335	3·6443344	4408·942	3·6061881	4038·203	3·4736521	2976·131
30	3·6228210	3·6254041	4220·890	3·6135672	4107·402	3·5824595	3823·486	3·4651445	2918·400
31	3·5838514	3·5869413	3863·145	3·5789461	3792·680	3·5541041	3581·825	3·4517607	2829·833
32	3·3492445	3·3527635	2253·012	3·3481032	2228·965	3·3289403	2132·751	3·2414517	1743·620
33	3·2870781	3·2909512	1936·725	3·2890603	1945·630	3·2748254	1882·984	3·2016263	1590·840
34	3·2110564	3·2152215	1641·427	3·2155729	1642·755	3·2054460	1604·893	3·1451302	1396·787
35	3·0895554	3·0939463	1241·500	3·0952446	1245·216	3·0896518	1229·283	3·0414036	1100·028
36	2·8107096	2·8152943	653·573	2·8191350	659·379	2·8156226	654·068	2·7783802	600·317
37	3·0129257	3·0176708	1041·525	3·0228197	1053·925	3·0218876	1051·690	2·9945491	987·527
38	2·2880255	2·2929142	196·297	2·2991582	199·140	2·3004183	199·718	2·2819951	191·423
39	2·2880255	2·2930146	196·343	2·3001663	199·603	2·3032706	201·034	2·2926864	196·194
40	2·7139943	2·7190748	523·691	2·7269967	533·331	2·7316508	539·077	2·7279671	534·524
		In feet	122545·996	....	97822·301	....	79994·644	....	51909·575
		In miles......	23·209	....	18·527	....	14·980	....	9·831
Multiply by 1″·1392, DEFLECTIONS = 26″·440					= 21″·106		= 17″·066		= 11″·199

Table of Deflections for Station B (*Kalianpur*) under the same circumstances.

No. of the compartment.	Depth =100 miles. log S.	Depth =100 miles. $\log \frac{c}{k}$ S.	Depth =100 miles. $\frac{c}{k}$ S.	Depth =300 miles. $\log \frac{c}{k}$ S.	Depth =300 miles. $\frac{c}{k}$ S.	Depth =500 miles. $\log \frac{c}{k}$ S.	Depth =500 miles. $\frac{c}{k}$ S.	Depth =1000 miles. $\log \frac{c}{k}$ S.	Depth =1000 miles. $\frac{c}{k}$ S.
22	2·0494119	2·0418442	110·114	1·9658756	92·433	2·8605227	725·307	1·6332740	42·981
23	2·8386287	2·8332533	681·167	2·7694447	588·091	2·6748921	473·034	2·4587111	287·548
24	3·3455110	3·3420835	2198·283	3·2891108	1945·856	3·2053158	1604·412	3·0013574	1003·130
25	3·5026319	3·5005562	3166·330	3·4565483	2861·200	3·3823463	2411·827	3·1904774	1550·520
26	3·5810559	3·5803537	3804·991	3·5452762	3509·750	3·4816049	3031·132	3·3045034	2016·060
27	3·6753989	3·6757504	4739·694	3·6478858	4445·144	3·5934441	3921·427	3·4308328	2696·701
28	3·7356791	3·7369177	5456·544	3·7152379	5190·842	3·6692980	4669·796	3·5215909	3323·463
29	3·7441811	3·7461551	5573·848	3·7297821	5367·625	3·6916358	4916·271	3·5590998	3623·262
30	3·7318654	3·7344485	5425·609	3·7226126	5279·740	3·6915039	4914·778	3·5741889	3751·361
31	3·6974044	3·7004943	5017·580	3·6924991	4926·053	3·6676571	4652·186	3·5653137	3675·477
32	3·6563681	3·6598871	4569·693	3·6552268	4520·920	3·6360639	4325·775	3·5485753	3536·513
33	3·6048058	3·6086789	4061·429	3·6067880	4043·785	3·5925531	3913·390	3·5193540	3306·390
34	3·5466938	3·5508589	3596·325	3·5512103	3558·036	3·5410834	3476·029	3·4807676	3025·294
35	3·4109670	3·4153579	2602·303	3·4166562	2610·095	3·4110634	2576·697	3·3628152	2305·766
36	3·3179997	3·3225844	2101·766	3·3264251	2120·435	3·3229127	2103·355	3·2856703	1930·502
37	3·1706547	3·1753998	1497·614	3·1805487	1515·475	3·1796166	1512·225	3·1522781	1419·967
38	2·9224140	2·9274031	846·064	2·9335467	858·117	2·9397959	870·554	2·9163836	824·866
39	2·8275631	2·8326436	680·211	2·8397039	691·359	2·8478887	704·512	3·8322240	679·554
40	......	......	......	......	......	......	......	......	......
		In feet	56129·565	......	54124·956	......	50802·707	......	38999·355
		In miles ...	10·631	......	10·251	......	9·6217	......	7·3862
Multiply by 1″·1392, Deflections=12″·111					=11″·678		=9″·622		=7″·386

Table of Deflections for Station C (*Damargida*) under the same circumstances.

No. of the compartment.	Depth =100 miles. log S.	Depth =100 miles. $\log \frac{c}{k}$ S.	Depth =100 miles. $\frac{c}{k}$ S.	Depth =300 miles. $\log \frac{c}{k}$ S.	Depth =300 miles. $\frac{c}{k}$ S.	Depth =500 miles. $\log \frac{c}{k}$ S.	Depth =500 miles. $\frac{c}{k}$ S.	Depth =1000 miles. $\log \frac{c}{k}$ S.	Depth =1000 miles. $\frac{c}{k}$ S.
30	2·1911155	2·1936986	156·206	2·1818617	152·006	2·1507540	141·499	2·0334390	108·004
31	3·3765405	3·3796304	2396·792	3·3716352	2353·072	3·3467932	2222·252	3·2444498	1755·698
32	3·5917478	3·5952668	3937·920	3·5906065	3895·888	3·5714436	3727·722	3·4839550	3017·580
33	3·6934978	3·6973709	4981·623	3·6954800	4959·981	3·6812451	4800·043	3·6080460	4055·515
34	3·6776695	3·6818346	4806·562	3·6521860	4810·453	3·6720591	4699·580	3·6117433	4090·190
35	3·6181590	3·6225499	4193·242	3·6238482	4205·800	3·6182554	4151·982	3·5700072	3715·413
36	3·5499407	3·5545254	3585·300	3·5583661	3617·145	3·5548537	3588·010	3·5176113	3293·149
37	3·4687918	3·4735369	2975·340	3·4786858	3010·827	3·4777537	3004·372	3·4504152	2821·080
38	3·3658939	3·3707826	2348·455	3·3770266	2382·470	3·3782867	2389·388	3·3598635	2290·148
39	3·1030610	3·1080501	1282·479	3·1152018	1303·772	3·1183061	1313·125	3·1077219	1281·510
40	3·0394934	3·0445739	1108·087	3·0524958	1128·485	3·0571499	1140·643	3·0534662	1131·009
		In feet	31772·006	......	31819·899	......	31178·616	......	27559·296
		In miles ...	6·017	......	6·027	......	5·905	......	5·2196
Multiply by 1″·1392, Deflections= 6″·855					=6″·866		=6″·670		=5″·220

1·8975923 and 1·9030900, 1·4090396 and 1·4259687, 1·1751240 and 1·2041200, 0·8406207 and 0·9030900. The values of θ are obtained from the first column in the Table of Heights (page 750), or from page 67 of my former Paper.

The values of $\frac{c}{k}$S for the three stations, for the various examples of depth, are now easily obtained by adding the logarithms of the last columns of the Four Tables of Reduction to the logarithms of S in the Table in page 753, and finding the natural numbers. The results are given for each of the three stations A, B, C, for all four cases of depth, in the four next Tables, which I call the Four Tables of Deflections in the meridian; the totals at the foot of these Four Tables give the final results, viz. the Deflections under the various suppositions of depth. These I now proceed to discuss.

§ 5. *Discussion of the Deflections under the various cases of depth, and of the effect upon the Ellipticity of the Indian Arc.*

13. The Tables thus calculated furnish the following results:—

	At Kaliana.	At Kalianpur.	At Damargida.
Deflection in meridian, caused by the mass of the Himmalayas and the mountain region beyond	27·978″	12·047″	6·790″
Ditto, by same mass distributed through a depth of . . . . 100 miles	26·440	12·111	6·855
Ditto, ... 300 miles	21·106	11·678	6·866
Ditto, ... 500 miles	17·066	9·622	6·670
Ditto, ... 1000 miles	11·199	7·386	5·220

By subtracting each of the last four lines from the first line, we have the following results:—

	At Kaliana.	At Kalianpur.	At Damargida.
Deflection in meridian, caused by the mass of the Himmalayas and of the mountain region beyond	27·978″	12·047″	6·790″
Ditto, modified by the supposed attenuation of matter extending down to a depth of 100 miles	1·538	−0·064	−0·065
Ditto, ... 300 miles	6·872	0·369	−0·076
Ditto, ... 500 miles	10·912	2·425	0·120
Ditto, ... 1000 miles	16·779	4·661	1·570

It will be seen how much the Deflections are reduced by this hypothesis, especially in the case where the attenuation extends through only 100 miles. In fact, in this case the upheaval of the mountains and the consequent attenuation below produce a slight deviation the other way at the two further stations. The success of the hypothesis may therefore, thus far, be considered to be established, although it remains an hypothesis still; and we must always be in uncertainty, not as to its answering this end, but as to its being true in nature. The existence of the mountain-mass is a fact indisputable. Not so the compensating cause, which is simply conjectural as to its existence, and altogether uncertain as to its extent, if it exist. We have no certain and independent method of determining this; nor of ascertaining, even if the hypothesis be valid, how far down the attenuation extends, or what law it follows.

14. I will now determine the effect of these several results upon the Ellipticity of the Indian Arc.

The corrections to be applied to the astronomical amplitudes of the two arcs Kaliana—Kalianpur and Kalianpur—Damargida, are determined by taking the differences of the numbers in each line of the Deflections above given. Hence we get—

	Kaliana to Kalianpur.	Kalianpur to Damargida.
Corrections to be added to the astronomical amplitudes, in consequence of the attraction of the mass of the Himmalayas and of the region beyond	$15''\cdot931$	$5''\cdot257$
Ditto, from the same cause, modified by the supposed attenuation below, extending down to a depth of 100 miles	1·602	0·001
Ditto, 300 miles	6·503	0·445
Ditto, 500 miles	8·487	2·305
Ditto, 1000 miles	12·118	3·091

Putting the arbitrary expressions $15''\cdot885\,(1-u)$ and $5''\cdot059\,(1-v)$, taken from par. 8, for these pairs of corrections in succession, we find the following corresponding values of u and v:—

$$u=-0\cdot0028 \quad \text{and} \quad v=-0\cdot0391$$
$$+0\cdot8992 \qquad +0\cdot9998$$
$$0\cdot5906 \qquad 0\cdot9120$$
$$0\cdot4657 \qquad 0\cdot5444$$
$$0\cdot2371 \qquad 0\cdot3890$$

By substituting these, in succession, in the formula (2) of par. 8 for ε, the ellipticity, we have the following values:—

$\varepsilon=0\cdot002377$ or $\frac{1}{421}$, when no Deficiency of Matter exists.

$0\cdot004621$ or $\frac{1}{216}$, when Deficiency of Matter extends down 100 miles.

$0\cdot003573$ or $\frac{1}{280}$, when Deficiency of Matter extends down 300 miles.

$0\cdot003497$ or $\frac{1}{286}$, when Deficiency of Matter extends down 500 miles.

$0\cdot002815$ or $\frac{1}{355}$, when Deficiency of Matter extends down 1000 miles.

The value of the semi-axis major may similarly be found. These values of the ellipticity differ in every instance from the mean value, $\frac{1}{300}$. They seem, however, to point out, that if the attenuation extend down to between 500 and 1000 miles the ellipticity would come out equal to the mean value. In this case the residual mountain attraction is considerable. Thus, then, it appears, that although the hypothesis of Deficiency of Matter, if it extend to no greater depth than 100 miles or so, will very much multiply the effect of the Himmalayas and the mountain-region beyond on the plumb-line, the result shows that in computing the Indian Arc there is little ground for working with the mean ellipticity, as is done in the Great Trigonometrical Survey.

15. Nor are the peculiarities of the Great Trigonometrical Survey explained by the hypothesis of Deficiency of Matter below. None of the pairs of errors in the astronomical amplitudes given in par. 14, nor any which can be interpolated, coincide with the errors in the amplitudes detected by Colonel EVEREST in the Great Arc*. According to him, astronomical observations make the amplitude of the arc Kaliana—Kalianpur too small by $5''\cdot236$, and that of the arc Kalianpur—Damargida too large by $3''\cdot789$. This latter error indicates the existence of some local cause of disturbance in the neighbourhood of Damargida. That such local causes may exist without the presence of so enormous a mass as that of the Himmalayas, I have shown in a Paper on the English Arc published in the 'Transactions' for 1856. For example, in the course of the calculations of that Paper the following instance occurs (p. 46, in the Table). A tabular mass only about 600 feet high, and measuring thirty-seven miles by forty-six, will cause a deflection of $3''\cdot172$ at a station three miles from its longer side, and about one-third from one end of it and two-thirds from the other end. Colonel EVEREST himself enters into an elaborate calculation in his earlier quarto volume (of 1830) to find the local attraction at the Station Takal K'hera, which had been previously fixed upon as one of the principal stations of the Great Arc; but was afterwards abandoned, notwithstanding all the time and labour which had been bestowed upon calculations in connexion with it; and that because the deflection $5''\cdot098$ was found to result from the attraction of an extensive table-land commencing as far as twenty miles off, and rising to only about 1600 feet above the level of the station. Colonel LAMBTON had considered the table-land to be too distant to have any effect; but calculation proved the reverse.

16. In fact, as it appears to me, while the absolute necessity of attending to local attraction is strongly illustrated by the investigations I have given in this and my previous Papers, the utter hopelessness of attaining to any certain results seems to be as clearly demonstrated. Here are causes, not only visible in mountain-masses and table-lands and deep neighbouring oceans which affect the vertical, and which can be calculated if sufficient labour be bestowed; but also invisible and hidden, as important as the others, of which we know absolutely nothing, except that they may exist and that we have no power of proving or disproving their existence. Were it not for these disturbing causes, the comparison of the parts even of the same arc ought to bring out its ellipticity (if the meridian *be* elliptical) with considerable precision. The approximation can

* See p. clxxvii of his 4to volume published in 1847.

be carried to any degree of exactness by the formulæ. The measures of the Survey give almost exact results. The lengths of the arcs measured are known to a wonderful degree of exactness. But the smallest disturbing causes affecting the plumb-line introduce an element of doubt and uncertainty, through the astronomical amplitudes, which vitiates the whole. This is unhappily the case with the Great Indian Arc. The errors brought to light in Colonel EVEREST's volume (of 1847), viz. $5''\cdot236$ in one portion and $-3''\cdot789$ in another, are too important not to be strictly and numerically accounted for. The cause *must* lie in the determination of the vertical. The deflection of the plumb-line caused by the attraction of the vast Mountain Region on the north will not account for these errors: it would make them very much larger (and would not explain the negative sign), as I showed in my Paper in 1855. No hypothesis of deficiency of matter below, which we can conceive, will remove the anomaly. The disturbing cause must lie elsewhere; perhaps in the immediate neighbourhood of Kalianpur and Damargida, which should be most carefully surveyed for the purpose of detecting it; or it may be hidden beneath the surface, and arise from sources which we cannot possibly examine and calculate, because of our ignorance of their position and magnitude. [This will be illustrated in the following paragraph.] Some would recommend that we ignore the above errors altogether, and put them to the account of these uncalculated causes. They would reverse the problem, and make the errors the measure of the disturbing cause. But unfortunately this will not do; for it proceeds upon the *assumption* that the meridian is elliptical, and moreover that its ellipticity is known and is equal to the mean ellipticity of the whole earth; which is, in short, begging the question at issue*.

* The degree of influence which errors in the amplitude and ellipticity may have in the mapping of the country may be learnt from the following approximate calculation.

Let A be the length of the arc, λ the amplitude, a the semi-axis major, μ the latitude of the middle point, ε the ellipticity of the arc, m the mean radius vector, then $m=a\left(1-\frac{1}{2}\varepsilon\right)$: also

$$\frac{A}{\lambda}=a\left\{1-\frac{1}{2}\varepsilon(1+3\cos 2\mu)\right\}=m\left\{1-\frac{3}{2}\varepsilon\cos 2\mu\right\} \quad \ldots \ldots \quad (\alpha.)$$

Suppose that the quantities used in this formula are incorrect, and that they ought to be $A+dA$, $\lambda+d\lambda$, $m+dm$, $\varepsilon+d\varepsilon$; the corrections are connected by the following equation:—

$$\frac{dA}{A}=\frac{d\lambda}{\lambda}+\frac{dm}{m}-\frac{3}{2}\cos 2\mu . d\varepsilon.$$

Now in the arc between Kaliana and Kalianpur, A is 371 miles $\left(\text{calculated on the supposition that the ellipticity }=\frac{1}{300}\right)$, λ is 19417, $\cos 2\mu=0\cdot59295$. I shall suppose that the *mean* radius m is correct, and therefore $dm=0$;

$$\therefore \quad \frac{dA}{A}=\frac{d\lambda}{\lambda}-0\cdot889 d\varepsilon;$$

$$\therefore \quad dA=\frac{A}{\lambda}(d\lambda-17270 d\varepsilon)=\frac{1}{52\cdot3}(d\lambda-17270 d\varepsilon) \text{ miles.} \quad \ldots \ldots \quad (\beta.)$$

In this formula $d\lambda$ must be put = to the deflection in seconds, with the *negative sign*, for the following

I proceed now to make use of the calculations of this Paper to show how some idea may be formed of the effect, which the possible and not improbable existence of extensive tracts within the mass of the earth where the density is, though perhaps very slightly, either greater or less than the density of those parts as required by the fluid theory of equilibrium.

§ 6. *General conclusions regarding the effect of a defect or excess of density in any part of the mass of the earth.*

17. It is quite possible that extensive tracts may exist in the interior of the earth's mass throughout which the density is somewhat less or somewhat greater than the density which the fluid theory would require for those parts, and upon which the theoretical direction of gravity is determined. I can conceive of a vast tract beneath in which the development of local heat had by long action expanded the material of the mass, and compressed it in a region beyond where no sufficient heat was developed to counteract this effect. Suppose this took place chiefly in a direction parallel to the

reason. A in formula (α.), the length of the arc between Kaliana and Kalianpur, is known accurately by the Survey: and, values of m and ε being assumed, λ is calculated by the formula. But λ comes out *larger* than the *observed* value, the difference arising from mountain attraction. Now $\lambda+d\lambda$ is used in formula (β.) to find how much A is affected by taking the *observed*, and not the calculated, value of λ. Hence $d\lambda$ must be negative. Let l be the number of seconds in the deflection; then $d\lambda=-l$, and

$$dA=-\frac{1}{52\cdot3}(l+17270d\varepsilon)=1\cdot1-\frac{l}{52\cdot3}-330\left(d\varepsilon+\frac{1}{300}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma.)$$

This formula agrees with the statement of Colonel EVEREST in p. clxxvii of the Preface of his volume of 1847: for by putting $d\varepsilon=0$ (as he calculates with the mean ellipticity), and $l=5''\cdot23$ the error he gives, $dA=-\frac{1}{10}$ mile $=-88$ fathoms, the error in the arc which he deduces. If then a place between Kaliana and Kalianpur be laid down on the map, first by reckoning from Kaliana and then from Kalianpur, the two spots will not coincide, but will be separated by $\frac{1}{10}$th of a mile. Colonel EVEREST avoided this by distributing the error among all the stations of the arc, and slightly altering all the latitudes accordingly. But the natural way of correcting it, as it appears to me, would have been to find an ellipticity which would have reduced the error to zero, and to have worked with *that* and not with the mean. In doing this, it would also be necessary, as shown in the calculations of these Papers, to correct for mountain attraction.

As a further illustration of the use of formula (γ.), and to show the influence of errors on the mapping part of the Survey, suppose that the deflections, after all, owing to the compensating cause, make l only $0''\cdot523$, and that the curvature of the arc is measured by the ellipticity $\frac{1}{400}$ (which is not impossible), then $d\varepsilon+\frac{1}{300}=\frac{1}{400}$, and $dA=1\cdot1-0\cdot01-0\cdot825=0\cdot26$, and the error in the places would be more than a quarter of a mile, and in the opposite direction.

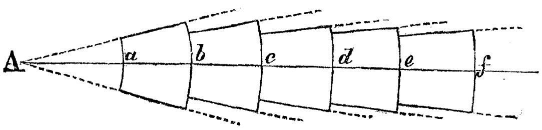
If the pairs of values of l and $d\varepsilon$ taken from par. 14 are put in formula (γ.) they will not reduce dA to zero, because they are mixed up with the data of the other arc, and are not correct, as is evident from the calculations of this Paper, as local causes of disturbance near Kalianpur or Damargida, or both, are not taken account of.

surface, so as to raise no considerable mountain mass. The effect on the plumb-line at a station over the place where the expanded and compressed portions meet would nevertheless be considerable.

I will proceed to calculate the effect on the plumb-line of a large given space (the space I use is 4 millions of cubic miles, *i. e.* half a cube of 200 miles each way, the thickness being 100 miles and always vertical), situated at different depths and distances from the station where the plumb-line is, and of a density equal to $\frac{1}{100}$th part of the density of the part of the earth where its middle point lies.

Let A*f* (fig. 3) be the meridian through A. Draw upon it five four-sided figures bounded by parts of great circles diverging from A, and also by circles of which A is the centre: and let their dimensions be so chosen, that the meridian length of each is 200 miles ($=2° 52' 40''$) and the area of each equals a square of 200 miles each way. It will result from this, that the angular distances of the sides of the spaces from A will be 4°, 6° 53′, 9° 46′, 12° 38′, 15° 31′, 18° 23′. By comparing these with the angular distances of the "compartments" by which I have before divided the meridian, it will be seen that the five "spaces" comprise the following whole and fractional parts of the "compartments:"—

Fig. 3.



Space *ab* includes $\frac{1}{2}$ of 18th, whole of 19th to 23rd, $\frac{1}{4}$ of 24th compartment.
Space *bc* includes $\frac{3}{4}$ of 24th, whole of 25th and 26th, $\frac{12}{13}$ of 27th compartment.
Space *cd* includes $\frac{1}{12}$ of 27th, whole of 28th and 29th, $\frac{6}{10}$ of 30th compartment.
Space *de* includes $\frac{4}{10}$ of 30th, whole of 31st , $\frac{2}{5}$ of 32nd compartment.
Space *ef* includes $\frac{3}{5}$ of 32nd . . . . $\frac{3}{7}$ of 33rd compartment.

The angular distances of the middle points of the five spaces from A are 5° 26′, 8° 19′, 11° 12′, 14° 4′, 16° 57′: and therefore the chords of these angles (rad.$=$4000 miles), or the values of c for the middles of the five spaces, are

379, 581, 781, 980, 1173 miles.

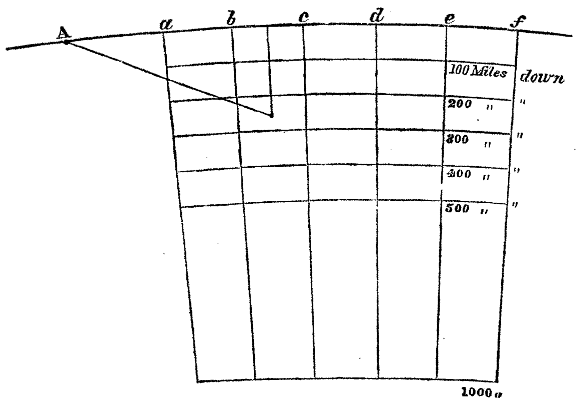
The angles of the lunes of which the five spaces are portions are found from the expression

$$\beta=\frac{\text{area of space}\times 180°}{2\pi r^2 \sin 1° 26' 20'' \sin(\text{angular distance of middle point from A})}.$$

The results are $\beta=30° 7'$, $19° 42'$, $14° 41'$, $11° 45'$, $9° 47'$.

18. Suppose now that each of the five spaces is covered by a mass one mile in uniform height; and that in each case this mass is then distributed uniformly to the three depths in succession, 100, 300, and 500 miles, as represented in fig. 4, which is a vertical section. I propose to find the deflection caused at A by the mass under these several circumstances. For this purpose we must use formula (3.) of paragraph 10. The calculation is much simplified, as h is always the same, and $=1$, and $Az=0$. The values of

Fig. 4.



$\log\left(1''{\cdot}1392\sin\frac{1}{2}\beta\right)$ for the five spaces are as in the margin. The values of $\frac{c}{k}$ belonging to each space must next be gathered out from the first, second, and third of the Tables of Reduction (see pages 754, 755), which give the values of $\log\frac{c}{k}$. The natural numbers are formed into the following Table, which gives the values of $\frac{c}{k}$ for such of the compartments (from the 18th

$\bar{1}{\cdot}4712930$
$\bar{1}{\cdot}2897722$
$\bar{1}{\cdot}1625924$
$\bar{1}{\cdot}0661096$
$\bar{2}{\cdot}9866678$

No. of compartment.	Values of $\frac{c}{k}$ gathered from the 1st, 2nd, and 3rd of Tables of Reduction (in par. 12) for following depths:—		
	100 miles.	300 miles.	500 miles.
18	0·95149	0·69152	0·49300
19	0·96131	0·72759	0·53054
20	0·97029	0·76186	0·56876
21	0·97676	0·79471	0·60814
22	0·98273	0·82502	0·64731
23	0·98770	0·85274	0·68590
24	0·99214	0·87821	0·72411
25	0·99523	0·89932	0·75808
26	0·99838	0·92092	0·79533
27	1·00081	0·93861	0·82803
28	1·00286	0·95402	0·85826
29	1·00456	0·96739	0·88604
30	1·00597	0·97892	0·91125
31	1·00714	0·98877	0·93380
32	1·00814	0·99738	0·95432
33	1·00896	1·00457	0·97218

to the 33rd) as are comprised in the five spaces. We must now apportion these values of $\frac{c}{k}$ to the several spaces to which they appertain, take their sum, and multiply by $1''{\cdot}1392\sin\frac{1}{2}\beta$, and we shall have the Deflections produced by the mass of one mile high and on a base equal to a square of 200 miles each way, when diffused downwards to the several depths of 100, 300, and 500 miles. The following Table will explain itself:—

Table, giving the Deflections caused by a Mass, which at the surface is 1 mile high and 40,000 square miles in base, diffused downwards to the following depths; viz.—

No. of compartment.	Compartments, in whole or in part, comprised in the spaces.	100 miles.	200 miles (by interpolation).	300 miles.	400 miles (by interpolation).	500 miles.
18	Half	0·47574		0·34576		0·24650
19	whole					
20	whole					
21	whole } space *ab* ...	4·87879		3·96192		3·04065
22	whole					
23	whole					
24	$\frac{1}{4}$th	0·24803		0·21955		0·18103
		5·60256		4·52723		3·46818
		log=0·7483865		log=0·6558326		log=0·5401015
		$\bar{1}$·4712930		$\bar{1}$·4712930		$\bar{1}$·4712930
		0·2196795		0·1271258		0·0113945
		Deflect.=1″·658	1″·499	1″·340	1″·183	1″·027
24	$\frac{3}{4}$ths	0·74411		0·65866		0·54308
25	whole } space *bc* ...	1·99361		1·82024		1·55341
26	whole					
27	$\frac{12}{13}$ths	0·92383		0·86641		0·76434
		3·66155		3·34531		2·86083
		log=0·5636649		log=0·5244364		log=0·4564920
		$\bar{1}$·2897722		$\bar{1}$·2897722		$\bar{1}$·2897722
		$\bar{1}$·8534371		$\bar{1}$·8142086		$\bar{1}$·7462642
		0″·714	0″·689	0″·652	0″·614	0″·558
27	$\frac{1}{13}$th	0·07698		0·07220		0·06369
28	whole } space *cd* ...	2·00742		1·92141		1·74430
29	whole					
30	$\frac{6}{10}$ths	0·60358		0·58735		0·54675
		2·68798		2·58096		2·35474
		log=0·4294260		log=0·4117812		log=0·3719430
		$\bar{1}$·1625924		$\bar{1}$·1625924		$\bar{1}$·1625924
		$\bar{1}$·5920184		$\bar{1}$·5743736		$\bar{1}$·5345354
		0″·391	0″·384	0″·375	0″·363	0″·342
30	$\frac{4}{10}$ths	0·40239		0·39157		0·36450
31	whole } space *de* ...	1·00714		0·98877		0·93380
32	$\frac{4}{10}$ths	0·40326		0·39895		0·38173
		1·81279		1·77929		1·68003
		log=0·2583474		log=0·2502467		log=0·2253170
		$\bar{1}$·0661096		$\bar{1}$·0661096		$\bar{1}$·0661096
		1·3244570		$\bar{1}$·3163563		$\bar{1}$·2914266
		0″·212	0″·210	0″·207	0″·203	0″·196
32	$\frac{6}{10}$ths } space *ef* ...	0·60488		0·59843		0·57259
33	$\frac{3}{7}$ths	0·43241		0·43053		0·41665
		1·03729		1·02896		0·98924
		log=0·0159000		log=0·0123985		log=$\bar{1}$·9953017
		$\bar{2}$·9866678		$\bar{2}$·9866678		$\bar{2}$·9866678
		$\bar{1}$·0025678		$\bar{2}$·9990663		$\bar{2}$·9819695
		0″·101	0″·100	0″·099	0″·098	0″·096

19. This Table furnishes the following results:—

	Distance of middle point from A, along the chord.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
Deflections, caused by the mass distributed downwards through a depth of ... 100 miles	$1''\cdot658$	$0''\cdot714$	$0''\cdot391$	$0''\cdot212$	$0''\cdot101$
Ditto .... 200 miles	1·499	0·680	0·384	0·210	0·100
Ditto .... 300 miles	1·340	0·652	0·375	0·207	0·099
Ditto .... 400 miles	1·183	0·614	0·363	0·203	0·098
Ditto .... 500 miles	1·027	0·558	0·342	0·196	0·096

Multiply these successive lines of numbers by 1, 2, 3, 4, 5, and we shall have the Deflections caused by masses having the same volumes as above, but all having the same density, viz. that of the first, *i. e.* $\frac{1}{100}$th part of the density of the materials of the surface. The numbers then become—

$1''\cdot658$	$0''\cdot714$	$0''\cdot391$	$0''\cdot212$	$0''\cdot101$
2·998	1·378	0·768	0·420	0·200
4·020	1·956	1·125	0·621	0·297
4·732	2·456	1·452	0·812	0·392
5·135	2·790	1·710	0·980	0·480

Now subtract each line from the line below it, and substitute the four lines thus formed for the last four above, and we have—

	Distance of the middle from A, measured along the chord.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
Deflections, caused by a semi-cubic mass, 200 miles in each horizontal side, and 100 miles deep, density $=\frac{1}{100}$th of the density of the surface, and the depth of its centre = 50 miles	$1''\cdot658$	$0''\cdot714$	$0''\cdot391$	$0''\cdot212$	$0''\cdot101$
Ditto .... 150 miles	1·340	0·664	0·377	0·208	0·099
Ditto .... 250 miles	1·022	0·578	0·357	0·201	0·097
Ditto .... 350 miles	0·712	0·500	0·327	0·191	0·095
Ditto .... 450 miles	0·403	0·334	0·258	0·168	0·088

The several volumes, the lower down they are, will, owing to the converging of the vertical lines, be somewhat contracted, and the densities slightly increased in a corresponding degree. If we suppose, therefore, the spaces to be correspondingly enlarged in the horizontal direction the volumes will be all the same, and the densities all the same; viz. $\frac{1}{100}$th part of the density of the surface, or of granite. But in order to compare them with the densities of those parts of the earth's interior where they are situated, we should increase their densities in proportion; and this will increase the deflection in a corresponding degree. The following is the usual law of density assumed from the fluid theory, D being the density of the surface, r = radius of the earth:—

$$\text{Density at depth } d \text{ miles} = \frac{r}{r-d} \mathrm{D} \sin\left(\frac{5\pi}{6} \frac{r-d}{r}\right),$$

from which I gather the following results:—

Ratio of density at depth 50 miles to D = 1·170
Ratio of density at depth 150 miles to D = 1·210
Ratio of density at depth 250 miles to D = 1·353
Ratio of density at depth 350 miles to D = 1·498
Ratio of density at depth 450 miles to D = 1·646

20. Multiply the lines in the last series of Deflections by these numbers, and we have our final results, as follows:—

Table of Deflections, caused by an excess or defect of matter throughout a semi-cubic space of 4 millions of cubic miles, the mean density of the excess or defect being $\frac{1}{100}$th part of the density of the earth at the depth of the centre of the cubic space.

Depth of the centre of the semi-cubic space.	Distance of the middle point of the space from A, measured along the chord to the surface.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
50 miles	1″·940	0″·835	0″·457	0″·248	0″·118
150 miles	1·621	0·803	0·456	0·252	0·120
250 miles	1·383	0·782	0·483	0·272	0·131
350 miles	1·067	0·749	0·490	0·286	0·142
450 miles	0·663	0·713	0·425	0·277	0·145

The effect of this calculation is to show how much uncertainty must always attend the exact determination of the true vertical, a thing which is absolutely essential in the calculation of the curvature of the several portions of the earth's surface. It will be observed that the supposed defect or excess of density has been assumed to be only $\frac{1}{100}$th part of the density of the earth where the hidden cause may lie. But a much larger fraction might have been chosen. Rocks at the surface of the earth, even of the same kind, differ considerably in their density according to the specimens examined. The following are examples:—

Basalt	varies through	$\frac{2}{9}$ths	of its mean density.
Chalk	varies through	$\frac{1}{6}$th	of its mean density.
Coal	varies through	$\frac{1}{4}$th	of its mean density.
Dolomite	varies through	$\frac{1}{9}$th	of its mean density.
Felspar	varies through	$\frac{2}{19}$ths	of its mean density.
Granite	varies through	$\frac{1}{8}$th	of its mean density.
Gypsum	varies through	$\frac{1}{5}$th	of its mean density.
Hornblende	varies through	$\frac{1}{6}$th	of its mean density.
Hornstone	varies through	$\frac{1}{13}$th	of its mean density.
Limestone	varies through	$\frac{2}{9}$ths	of its mean density.

These show how probable it is that the several portions of the earth's interior, although preserving roughly the general average of density, according to their position, as required by the fluid theory, may nevertheless vary sufficiently to disturb the position of the plumb-line most materially. The space I have selected as the basis of my calculation is no doubt extensive; but I hardly think too extensive. I have seen the same kind of rock (gneiss) prevail for hundreds of miles in the Himmalaya Mountains; and can see no reason why a space as large as I have chosen, 200 miles square parallel to the surface and 100 miles deep, may not exist beneath the surface, having a density, too, differing much more than $\frac{1}{100}$th part from the proper density of its locality.

§ 7. Appendix, *containing a revise of some parts of my former Paper.*

21. The results of this Paper may appear in some respects to render the calculations of my former communication now unnecessary, as it is here shown that it is not improbable that a compensating cause exists sufficient to counteract the effect of the Mountain Mass, at any rate to a considerable degree. But it must be observed that the demonstration of the sufficiency of this cause, should it exist in nature, rests altogether upon the process of dissection of the mass and the calculations consequent thereon, given at large in that Paper. The conclusions also in the present communication regarding the effect of wide-spread, though minute, defect or excess of density below, rest upon those former calculations. I have thought it well, therefore, to take this opportunity of revising some parts into which errors have crept, as intimated in the Note to par. 1.

22. The corrections of the Deflections in meridian, produced by the Mountain Mass as it exists on the surface, at the three principal stations, have already been given. They are but trifling. I have, since completing this new calculation, gone over the former one and detected the several minute errors, so as to account for every figure of discrepancy*. This is highly satisfactory, especially when the troublesomeness of the calculation is considered.

* The errors have crept in among the calculations at the foot of the Six Tables of my former Paper (pp. 78–83). The details are as follows. The values of $\mathrm{H}\sin\frac{1}{2}\beta\cos\mathrm{Az.}$ as there given, and the necessary corrections, are as below:—

	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.	Totals.
For Station A. (miles)	0·772	3·656	4·262	2·602	0·095	
Errors ...	......	+0·001	−0·001			
	3·009	3·933	3·395	2·493	0·235	
Errors ...	+0·078	+0·001	......	+0·001	+0·027	
						24·559 miles.
For Station B.	0·015	1·600	1·211			
Errors ...	......	−0·046	−0·001			
	0·677	2·536	2·643	1·813	0·011	
Errors ...	+0·001	......	+0·020	+0·001	+0·094	
						10·575 miles.
For Station C.	1·173					
Errors ...	−0·124					
	1·973	2·103	0·816			
Errors...	......	+0·019				
						5·960 miles.

These three final quantities are precisely the same as at the foot of the Table in page 753.

In my former Paper I brought out the Deflections $27''{\cdot}853$, $11''{\cdot}968$, $6''{\cdot}909$
The correct values are now shown to be . . . . $27''{\cdot}978$, $12''{\cdot}047$, $6''{\cdot}790$

Determination of the Mass of the whole Mountain-region of the Enclosed Space.

23. The calculation on this subject needs also revision. The direct way of determining the volume of this mass is to find its average height and multiply it by the area of the Enclosed Space. First, then, I will find this area. By examining the diagram in par. 6 (fig. 1), it will be seen that by re-arranging some of the portions furthest from A the area of the Enclosed Space is equivalent to the part of a lune about 132° wide and stretching from A to the end of the 35th compartment, the angular distance of which is 21° 24′. Hence, by a known trigonometrical formula,—

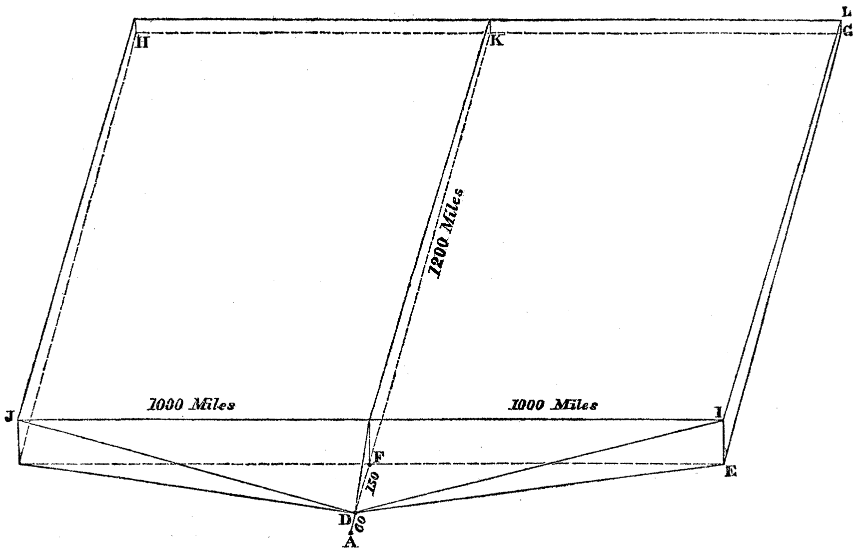
$$\text{Area of Enclosed Space} = \text{area of this portion of the lune}$$
$$=\frac{132}{180}\pi(1-\cos 21^\circ\, 24')r^2=2{,}559{,}162 \text{ square miles.}$$

24. We must now find the average height of the mass standing on this space. This is not at first sight an easy matter. We cannot obtain it by taking the average of the heights given in the Table of Heights in page 750, because the bases on which these heights stand are of such unequal extents, that an undue advantage would be given to those which stand upon the smaller bases. In my former Paper I overcame the difficulty by finding how much I must cut down the mass in order to reduce the attraction to zero; this quantity I considered to be the average height. It is obvious, that, as a general rule, this would not be true. It so happens, however, that for a mass shaped as the mass under consideration is, it is true. This I did not show in my former Paper, although I made use of the property. I propose now to supply the deficiency. The method I shall pursue is this. I take a geometrical figure which sufficiently well represents the actual mass in general form, but one of which the attraction upon A can be accurately calculated. I then show, in the case of this figure, that the average height obtained in the direct way by geometry, and also by the method of attractions, is the same. I infer, therefore, that it is so in the case of the actual mountain mass. The reason of this coincidence it is not difficult to see. The highest part of the mass is much nearer to A than the middle of the mass is. Suppose the highest parts had been about the centre; then in levelling these down so as to form the table-land which would have the average height, equal portions would have to be brought to equal distances from the middle towards A and opposite to A. The latter transfer would add more to the attraction than the other would detract from it; and therefore the average volume would not be the average mass measured by the attraction. But as the higher parts of our mass are much nearer A than the centre, it is obvious that an exact compensation is possible. The calculation shows that it is real.

25. The accompanying diagram (fig. 5) represents the geometrical figure I have alluded to. It is drawn upon a scale, except that the vertical heights are exaggerated sixty times. A is the station Kaliana, D the point north-north-east where the mass begins, and thence

shelves up in a pyramidal form to the ridge IJ, 2000 miles long and 210 miles from A, and rising to an elevation above A of 1·5 mile; whence it shelves down again to GH,

Fig. 5.



1200 miles further off, where it abruptly terminates at a height of half a mile above the level of A. That this is a fair representative of the mass with reference to its effect on A, will be seen in the course of the investigation in this,—that its area, its volume, and its attraction on A, as well as its principal heights, are all the same as in the case in nature. The following are the measures: AF or $a=210$ miles; FE or $c=1000$ miles; FK or $e=1200$ miles; DF or $b=150$ miles; EI or $h=1·5$ mile; GL or $h'=0·5$ mile.

The area of the base of this figure $=2000(1200+75)=2{,}550{,}000$ square miles. The volume $=\frac{1}{2}(h+h')2000\times1200+\frac{1}{3}h\times2000\times150=2{,}550{,}000$ cubic miles. The average height of the whole figure $=\frac{\frac{1}{2}(h+h')2000\times1200+\frac{2}{3}h\times2000\times75}{2000\times1200+2000\times75}=1$ mile.

It is not difficult to prove by the Integral Calculus, that the Attraction of this mass on A in the direction AD, its density being half the mean density of the earth,

$$=\frac{g}{16760}\left\{\left\{h+\frac{a}{e}(h+h')\right\}\log_e\left[\frac{\sqrt{(a+e)^2+c^2}-c}{\sqrt{a^2+c^2}-c}\,\frac{a}{a+e}\right]-\frac{c}{e}(h-h')\log_e\left[\frac{\sqrt{(a+e)^2+c^2}+a+e}{\sqrt{a^2+c^2}+a}\right]\right.$$

$$\left.+h\,\frac{a-b}{c}\,\frac{\sqrt{1+A^2}-B}{B^3}-h\,\frac{a-b}{b}\,\frac{2B^2-1}{B^3}\log_e\left[\frac{A+\sqrt{1+A^2}}{\frac{b}{c}+B}\right]+h\,\frac{a-b}{b}\log_e\left[\frac{\sqrt{1+A^2}+\frac{c}{a-b}B}{\frac{a}{a-b}B}\right]\right\},$$

where $A=\frac{ab+c^2}{c(a-b)}$, $B^2=1+\frac{b^2}{c^2}$.

Also the attraction of a tabular mass on the same base and of height k

$$=\frac{g.k}{16760}\left\{\log_e\left[\frac{\sqrt{(a+e)^2+c^2}-c}{\sqrt{a^2+c^2}-c}\,\frac{a}{a+e}\right]+\frac{1}{B^3}\log_e\left[\frac{A+\sqrt{1+A^2}}{\frac{b}{c}+B}\right]-\log_e\left[\frac{\sqrt{1+A^2}+\frac{c}{a-b}B}{\frac{a}{a-b}B}\right]\right\}.$$

Putting the numerical values in these formulæ, we obtain

$$A=17\cdot175,\ \sqrt{1+A^2}=17\cdot204,\ B=1\cdot011;$$

and the final results are as follows:—

$$\text{Attraction on } A=0\cdot0001557g=\tan(32'')g.$$

Hence the Deflection of the plumb-line $=32''$, or, in the direction of the meridian, $=32''\cos 30^\circ=27''\cdot7$, 30° being about the azimuth of the line along which the whole attraction on A acts (see my former Paper). This deflection very nearly coincides with the deflection calculated in this Paper, which gives a further testimony that the actual mass, in reference to its effect on A, is fairly represented by our geometrical figure.

The last formula gives, Attraction of tabular mass on the same base $=0\cdot0001563\,g\,.\,k$. That this may be the same as the attraction of the geometrical figure, we must have

$$0\cdot0001563\,k=0\cdot0001557, \text{ or } k=0\cdot996 \text{ mile.}$$

But I have already shown that the average height of the geometrical figure is one mile, and is therefore almost exactly the same as this deduced from attraction. Hence the principle of determining the average height in this manner, in the case of a mass formed like the Himmalayas and the Mountain region beyond, is correct; and I shall proceed to use it to determine the average height more precisely.

26. If all the numbers in the five columns which appertain to Station A in the Table of Heights (page 750) were replaced by 1000, and the process were gone through by which the first column (that for A) in the Table of page 753 is formed, we should have the meridian deflection caused by a tabular mass standing on the enclosed space and 1000 feet in height above the level of A. This process leads to the following result. As the five numbers in each line of the Table from left to right are to be multiplied by the same five numbers, viz. by 0·1852, 0·2588, 0·2241, 0·1294, 0·0151, we may add up the columns first and then afterwards multiply the aggregates by these constants.

The aggregates are	27,000	32,000	36,000	40,000	32,000	feet.
Multiply these by	0·1852	0·2588	0·2241	0·1294	0·0151	
The products are	5000·4	8281·6	8067·6	5176·0	483·2	feet.
The same reduced to miles .	0·947	1·568	1·517	0·980	0·092	mile.

The grand total $=5\cdot104$ miles, and multiplied by $1''\cdot1392=5''\cdot814$.

This is the meridian deflection at A caused by a mass 1000 feet high.

Hence the average height of the whole mass

$$=\frac{27''\cdot978\times1000}{5''\cdot814}=4812 \text{ feet }=0\cdot911 \text{ mile.}$$

And the volume of the whole mass $=$area of Enclosed Space $\times$ average height

$$=2{,}550{,}000\times0\cdot911=2{,}323{,}050 \text{ cubic miles.}$$

Also the mass, taking the density ρ equal to half the mean density of the earth,

$$=\frac{3}{8\pi}\,\frac{2{,}323{,}050}{64{,}000{,}000{,}000}\times\text{Mass of the Earth}=\frac{1}{230{,}895}\text{th of Mass of the Earth.}$$

27. As I have not again calculated in this Paper the *total* deflections of the plumb-line at the three stations A, B, C, but only the *meridian* deflections, I cannot revise the positions of the three points where the whole mass must be collected that it may produce the same effects as in nature. This is of no importance. The principle devised for interpolating the deflection at any intermediate station between A and C, by means of the property of a curve, still holds good: and the amount of meridian deflection may be represented nearly by the expression $\frac{111''\cdot 7}{l-L+4}$, l and L being the latitudes of Kaliana and of the intermediate station on the arc between its extremities where the deflection is sought. It makes the deflection at the southern extremity about one-fifteenth too large; but at the northern and middle principal stations it gives it correctly*.

28. In the last page of my former Paper I compare the curvature of the Indian Arc under several hypotheses by means of the formula,

Height of the middle point of an arc of which the amplitude is λ, above the chord of the arc,

$$=\frac{1}{8}a.\lambda^2\left\{1-\varepsilon\left(\frac{1}{2}+\frac{3}{2}\cos 2\mu\right)\right\},$$

μ being the latitude of the middle point, a the radius of the earth, and λ sufficiently small to allow λ^4 to be neglected. This formula is correct. But I should not have left it in terms of λ, the amplitude, but of s, the length of the arc; since λ is not the same, whereas s is, in the three cases to which the formula is applied. This change will make the height above the chord

$$=\frac{s^2}{8a}\left\{1+\varepsilon\left(\frac{1}{2}+\frac{3}{2}\cos 2\mu\right)\right\}=20(1+1\cdot 512\varepsilon)\text{ miles},$$

which is the same as before, except in the sign of ε. The consequence of this is, that the arc is *flatter* when attraction is taken account of, and is *more curved* when it is neglected, than the mean curvature.

* The law of the inverse chord will naturally deviate from the truth, and give too large a value, as we recede from the Himmalayas, for the following reason. The Himmalayan Mass has been shown to produce the same effect as a comparatively slender uniform prism of great length running nearly east and west. Now the attraction of such a prism on a point opposite to its middle, equals its mass divided by the product of the point's distances from the middle and from either extremity of the prism. Hence when the distance from the middle, compared with the prism's length, is small, the attraction will vary most nearly as the inverse distance; but as the distance increases, the law evidently tends towards that of the inverse *square*, which it ultimately attains when the distance is very great compared with the length of the prism. This sufficiently accounts for the actual deflection at Damargida being somewhat smaller than that given by the formula, and tends therefore to confirm the general calculation.

Calcutta, September 1, 1858.

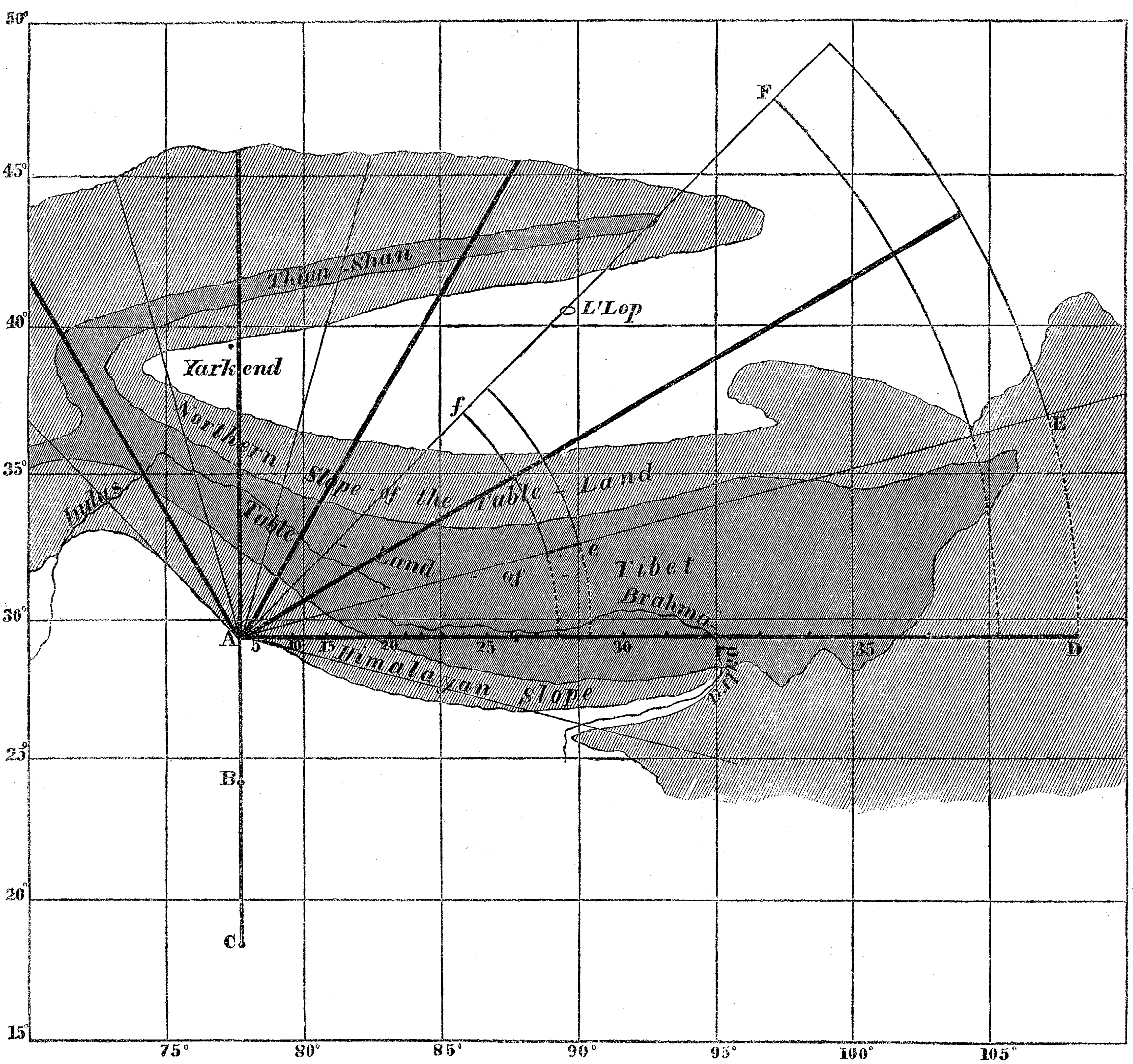
Received February 21, 1859.

Postscript on Himmalayan Attraction.

1. Since transmitting in September last my Paper "On the Deflection of the Plumb-line in India caused by the Attraction of the Himmalaya Mountains," &c., I have had the advantage of seeing the pages of a very interesting and valuable work by Major R. STRACHEY, now passing through the press, upon the Physical Geography of the Himmalayas; and being allowed to make use of them, I gladly avail myself of the important information they contain to add a Postscript to my former communications.

2. The new information now obtained gives a view of the regions which lie beyond the Himmalayan crest, differing in some respects from that which I gathered from the sources of scanty data which I have hitherto been able to consult. The results of Major STRACHEY'S investigations, combined with those of his brother Captain H. STRACHEY, are represented in the following diagram. A, B, and C (as in my former Papers) are the

Fig. 6.



three principal stations of the northern portion of the Indian Arc of Meridian. Kaliana

(29° 30′ 48″ lat.), Kalianpur (24° 7′ 11″), and Damargida (18° 3′ 15″). The parts of the diagram left white in the neighbourhood* of the range are the plains; the lightly shaded parts are the mountain slopes, with very varied surface, by degrees attaining the height of the celebrated plateau from which the Indus and the Brahmaputra take their rise and flow towards the Ocean on different sides of India; the darkly shaded parts are this plateau or table-land.

"The loftiest points known on the surface of the earth are to be found along the southern border of the Table-land among the mountains of the Himmalayan Slope; one of them having been measured the height of which exceeds 28,000 feet, while peaks of 20,000 feet abound along the entire chain. The plains of India which skirt the foot of its southern face for an extent of 1500 miles, nowhere have an elevation exceeding 1200 feet above the sea, the average being much less; and we have every reason to suppose that the northern plateau of Yarkend and Khotan, like the country around Bukhara, lies at a very small elevation, probably not more than 1000 or 2000 feet above the sea, while the surface, as we know, descends on the borders of the Caspian to 80 feet below that level" (pp. 4, 5). Major STRACHEY thinks that "none of the numerous ranges commonly marked on our maps of Tibet have any special definite existence as mountain chains, apart from the general mass of the Table-land, and that this country should not be considered to lie as if in the interval between the two so-called chains of the Himmalaya and Kouenlun, but that it is in reality the summit of a great protuberance above the general level of the earth's surface, of which the supposed Kouenlun and Himmalaya are nothing more than the north and south faces, while the other ranges are but corrugations of the table-land more or less strongly marked" (pp. 5, 6). The two rivers, Indus and Brahmaputra, "maintain a course along the length of the summit of the Table-land, and receive, as they proceed, the drainage of its entire breadth; with the exception, first, of an occasional strip along its southern edge, from which the water passes off more or less directly to the north through the Himmalaya; and, secondly, of some parts chiefly found in the northern half of the Table-land, from which the water has no escape, but is collected in lakes in depressions on its very summit. The waters thus accumulated in these two streams are at length discharged by two openings in the Himmalaya Slope through the plains of Hindostan into the Indian Ocean. None of the drainage of the table-land, so far as we know, passes in the opposite direction through the northern slope; and the area that discharges itself southward at points intermediate between the debouche of the Indus and Brahmaputra is, with one exception, that of the Sutlej, comparatively insignificant. The waters of the northern slope itself exclusively flow down to the plains of Yarkend; while in like manner those of the southern slope, with the drainage of the exceptional area along the southern border of the table-land running off to the south, traverse the Himmalaya more or less directly, and constitute such rivers as the Jumna, Ganges Proper, &c., and other main tributaries of the Indus, Ganges, and Brahmaputra" (pp. 32, 33).

* Down towards B and C the country becomes hilly; but not sufficiently so to affect my results.

Major STRACHEY, after his brother, calls the northern boundary of the table-land the Turkish Watershed, and the southern the Himmalayan Watershed (p. 33).

"The average elevation of the crest of the Indian watershed most probably exceeds 18,000 feet. In a comparatively few points only its continuity is broken, and it allows the passage of rivers that rise on its northern flank; but at all other points its summit must be crossed in entering Tibet from the south. The passes over it are frequently more than 18,000 feet above the sea; and, except where it is broken through, I know of no point to the east of Kashmir where it can be surmounted under 16,400 feet" (p. 51).

"The summit of the table-land, though deeply corrugated with valleys and mountains in detail, is in its general relief laid out horizontally, at a height little inferior to that of its southern scarp" (p. 52).

The plain along the upper course of the Sutlej "lies immediately to the north of the British provinces of Kumaon and Gurhwal, and is about 120 miles in length, its breadth varying from 15 to 60 miles. Its surface, to the eye a perfect flat, varies in elevation from 16,000 feet along its outer edges on the south-west and north-east, to about 15,000 feet in its more central parts, where it is cut through by the river Sutlej which flows at the bottom of a stupendous ravine, furrowed out of the alluvial matter of which the plain is composed to a depth of 2000 or 3000 feet, and at its west end even more" (p. 53). This will account for the statement on the Survey Map (as noticed at p. 75 of my Paper of 1855), at the point where the Sutlej leaves the table-land, that the height of its bed is only 10,792 feet. This I have taken in my former calculation as the *greatest* height of any of the compartments into which I divide the surface; this, therefore, the researches of Major STRACHEY show to be much under the mark.

On a careful consideration of all the data, Captain H. STRACHEY estimates the mean elevation of the table-land between the Himmalayan and Turkish watersheds, and to the west of the ridge between the sources of the Indus and Brahmaputra, to be 15,000 feet (p. 56).

3. A comparison of Major STRACHEY'S map, copied in part in the diagram above, with the diagram I have given in former Papers of the "Enclosed Space*," will show that much attracting matter which, from HUMBOLDT'S account, I supposed to exist, in Major STRACHEY'S description does not appear, at any rate not in so important a degree. Lest, therefore, it should be thought that, my data being in some respects wrong, my results are altogether vitiated, I have examined the effect of these new measures, and I find that the increased height given to the plateau compensates for the removal of any attracting mass higher north which I had supposed to exist.

4. I do not intend to enter anew into a lengthened calculation of the deflection of the plumb-line, with a view, as before, to obtain an exact result, because my object will be equally answered by taking a simpler course. My object in these various communications has been, first, to give an easy method of determining the amount of attraction

* Philosophical Transactions, 1855, p. 76.

and deflection, when the heights are known—this has been done in the Paper of 1855; secondly, to point out, from such trustworthy data as I could procure, that the amount of deflection is so great as to render it absolutely necessary to allow for it in finding the astronomical amplitude for geodetic operations; and, lastly, to suggest that such surveys and calculations should be made as to make it possible to determine the amount with a sufficient degree of precision.

The only calculation I propose now to make, is to show that the deflection caused by this Table-land *alone*, as laid down by Major STRACHEY, produces an error double the error brought to light by the Survey, in which mountain attraction is neglected and the ellipticity of the Indian arc is assumed to be the mean ellipticity of the whole earth. Much greater, then, will be the discrepancy, as I might easily show were it worth while, when the attraction of all the slopes—especially the parts nearest to A—is added.

5. Through A I have drawn a straight line AD, and have marked off certain divisions which indicate the Law of Dissection according to which the attracting mass is to be divided. From A several fainter lines are drawn dividing the attracting mass into lunes of 30° width: the dark lines bisect these lunes, one of them being in the meridian of A. Now if through the points of division of AD circles be drawn about A, they and the lunes will divide the surface into a number of four-sided compartments, like EF and *ef*: and the law of this dissection is so chosen, that if the height of the attracting matter on EF and *ef* were the same, the attraction of these two partial masses on A along the dark mid-line of the lune would be the same: this may also be expressed by saying, that the attraction of the mass on any compartment thus formed is proportional to the height of that mass. In a former Paper the following formula has been proved:—

Meridian deflection of plumb-line at A, caused by the attraction of a mass of height h miles standing on any compartment,

$$h \sin 15° \times 1''\cdot 139 \cos \text{azimuth of mid-line of the lune.}$$

6. I propose now to apply this formula to find the deflections caused at the three stations by the attraction of the Table-land alone. The height of the Table-land above Kaliana I will take to be $2\frac{2}{3}$ miles; that is, about 14,000 feet, or 15,000 feet above the sea. Then the above coefficient becomes $2\frac{2}{3} \times \cdot 2588 \times 1''\cdot 139 = 0''\cdot 786$; and

$$\text{Merid. Deflection} = 0''\cdot 786 \cos \text{azimuth.}$$

The calculation is rendered easy by the heights of all the compartments being the same; and the only difficulty is in finding how many compartments in each lune lie on the Table-land. This is done in the following manner:—A strip of paper is laid on the diagram along AD, and the divisions of AD marked upon it. This scale is then laid along the mid-line of each lune, and the number of divisions (and therefore the number of the compartments) which fall on the Table-land in that lune easily read off. In some instances the Table-land only partially fills a compartment; in that case compensation is made by diminishing the number of the compartments, in the ratio of the deficiency to the whole space of the compartment. This is indicated in the following Table by an *asterisk.* Lune V. is omitted in station A because it can have but very little effect in the meridian at A.

	Station A.				Station B.			Station C.	
	Lunes I.	II.	III.	IV.	Lunes II.	III.	IV.	Lunes II.	III.
No. of compartments on Table-land...	8* *or* 4	8	9	12	5	5	10* *or* 7·5	2	5* *or* 4
Cos azimuth	·866	1	·866	·5	1	·866	·5	1	·866
No. of equivalent compartments if placed in the meridian	3·464	8	7·794	6	5	4·33	3·75	2	3·464
Sums of these............................	25·258				13·08			5·464	
Multiply by 0″·786 and the Deflections are	19″·85				10″·28			4″·29	

Hence the errors produced in the astronomical amplitudes will be 9″·57 and 5″·99; which much exceed the errors 5″·236 and −3″·791 brought to light by the Survey, and will far more so when the attraction of the nearer parts is also taken into account.

7. The new information regarding the nature of the country immediately north of the Himmalayas, does not, it thus appears, relieve this subject of its difficulties: and no geodetic calculations can be of service in the problem of the Figure of the Earth, nor indeed in mapping the country with extreme precision, till these perplexities are removed, by the deflection being found and allowed for.

Calcutta, January 14, 1859.